Theory of Computer Science

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Exercise Sheet 8

Due: Wednesday, April 22, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 8.1 (Turing Machines; 1.5 Points)

Describe (in words) a Turing machine which computes the function $f(a^i) = b^{2i}$ for words over the alphabet $\Sigma = \{a, b\}$.
Note: Use statements which have a similar level of detail as the following examples: "move the reading head to the left, until it reads an a" and "if a b is read, go into an endless loop".

Exercise 8.2 (Definition of Turing Machines; 2 Points)

Specify the complete definition of a Turing machine which computes the *predecessor function* $pred_2$ over natural numbers (see slide 23 in slide set 13). You can specify the transition function δ as a state diagram. *Additionally* describe (in words) how your Turing machine works.

Exercise 8.3 (Computability of Compositions; 2 Points)

Let $f : \Sigma^* \to \Sigma^*$ and $g : \Sigma^* \to \Sigma^*$ be Turing-computable partial functions for an alphabet Σ. Show that the *composition* $(f \circ g) : \Sigma^* \to \Sigma^*$ is also turing-computable.
In general the composition of two functions is defined as $(f \circ g)(x) = f(g(x))$. Specifically, the value $(f \circ g)(x)$ is undefined if $g(x)$ is undefined.

Exercise 8.4 (Syntactic Sugar for LOOP-Programs; 3 Points)

Simulate the following syntactical constructs for LOOP-programs (with obvious semantics) by using already known constructs. This means you may use the base constructs of LOOP programs as well as the additional constructs introduced in section 14.2 in the lecture.

(a) **IF** $x_i < c$ **THEN** P **ELSE** P' **END**

(b) **IF** $x_i = x_j$ **THEN** P **END**

(c) **FOR** $x_i = 0$ **TO** c **DO** P **END**

In the above constructs P and P' are arbitrary LOOP-programs and $i, j, c \in \mathbb{N}_0$ are arbitrary natural numbers.

Exercise 8.5 (WHILE-Programs; 1.5 Points)

Which unary function does the following WHILE-program compute? Is this function LOOP-computable? Justify your answer to the latter question.

```
x2 := 1;
x3 := 0;
WHILE x2 ≠ 0 DO
  IF x1 = x3 THEN
    x2 := 0
  END;
  x3 := x3 + 2
END;
x0 := 1
```