Foundations of Artificial Intelligence

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Exercise Sheet 8

Due: April 26, 2017

Exercise 8.1 (2+1 marks)

Consider the constraint network that is given by the graph coloring problem of a graph $G = \langle V, E \rangle$. The set of vertices V contains a vertex for each Swiss canton, and E is such that two vertices v and v' are connected iff the cantons v and v' share a border. A description of G can be downloaded from the website of the course.

(a) Provide a cutset $V' \subseteq V$ for G that is as small as possible (it is not necessary to provide an explanation how you have found V'). As a reminder, a cutset of a graph is defined as a set of vertices that is such that the induced subgraph that is obtained by removing these vertices results in an acyclic graph.

Note: You get 2 marks for your solution if your cutset is optimal, 1 mark if your cutset contains exactly one more vertex than an optimal cutset and 0 marks otherwise.

(b) Assume we are interested in coloring G with 4 colors. Provide a worst-case estimate of the runtime of the algorithm based on cutset conditioning if your cutset of exercise 8.3 (a) is used (i.e., compute an upper bound for the number of considered assignments). Compare your result to the estimated runtime if no cutset is used.

Hint: It is simpler to solve this exercise if a graph visualization tool like, for instance, *graphviz* is used. The description of G that can found on the website of the course is formatted for graphviz. If you use Linux, you can create a pdf visualization of the graph with:

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dot -T pdf -o cantons.pdf cantons.dot
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Exercise 8.2 (0.5+0.5+0.5+0.5 marks)

Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions that clearly express what they encode.

(a) If it is warm and it is summer, Bob goes for a swim.

(b) If it rains, it is not warm or it is summer.

(c) If Bob is going for a swim, then it is always summer and he does not drink tea.

(d) Either Bob drinks tea or he is going for a swim (but not both).

Exercise 8.3 (2 marks)

Consider the propositional formula

$$\varphi = ((A \lor B) \to C).$$

Check for each of the following properties of propositional formulas if φ has that property. Justify your answers. Where possible, provide a suitable interpretation.

(a) satisfiable

(b) falsifiable

(c) valid

Exercise 8.4 (3 marks)

Consider the following syntax extension for propositional logic: $\psi \leftrightarrow \eta$ is a propositional formula if ψ and η are propositional formulas. Furthermore, let the semantics of $\leftrightarrow$ be defined by

$$I \models \psi \leftrightarrow \eta \text{ iff } I \models \psi \rightarrow \eta \text{ and } I \models \eta \rightarrow \psi.$$

Now consider the propositional formula

$$\varphi = ((A \vee B) \rightarrow C) \wedge \neg((C \vee B) \leftrightarrow A)$$

and the interpretation

$$I = \{A \mapsto \mathbf{F}, B \mapsto \mathbf{T}, C \mapsto \mathbf{T}\}.$$

Use the definition of $\models$ to show that $I \models \varphi$.

Exercise 8.5 (2 marks)

Use the semantics of propositional logic to show that for arbitrary formulas φ and ψ we have that

$$\varphi \rightarrow \psi \equiv \neg\varphi \vee \psi.$$

Important: The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.