Theory of Computer Science

G. Röger | University of Basel
Spring Term 2018 | Computer Science

Exercise Sheet 2

Due: Wednesday, March 14, 2018

Exercise 2.1 (Syntax; 0.25+0.25+0.25+0.25 Points)

Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions. Take care to fully parenthesize all formulas.

(a) If it does not rain, the sun is shining.

(b) If Bob is going for a swim, then the sun is shining and it is warm.

(c) Bob eats ice cream exactly if he is going for a swim or if it is warm and it does not rain.

(d) Either the sun is shining or it is raining (but not both).

Exercise 2.2 (Semantics; 0.25+0.25+1+1+0.5 Points)

Consider the following formula:

$$\phi = ((A \vee (((\neg B \vee C) \wedge (C \leftrightarrow \neg D)) \vee (D \rightarrow E))) \rightarrow (F \rightarrow \neg A))$$

(a) How many lines would be needed for a truth table for ϕ?

(b) Formula ϕ is an implication. Specify the truth table for the general implication formula $\varphi \rightarrow \psi$. Attention: You should **not** specify the truth table of ϕ.

(c) Specify a model $\mathcal{I}$ for ϕ and prove without truth table that $\mathcal{I} \models \phi$.

(d) Specify an assignment $\mathcal{I}$ with $\mathcal{I} \not\models \phi$ and prove that $\mathcal{I}$ has the desired property.

(e) Which of the properties *satisfiable*, *unsatisfiable*, *valid*, and *falsifiable* are true for ϕ? Justify your answer for each of the four properties.

Hint: The proofs for this exercises are fairly short (4 and 6 steps, respectively). If you need a considerably larger amount of steps, rethink your solution and try to find an easier proof. The solution of part (b) may help you identify the requirements for $\mathcal{I}$.

Exercise 2.3 (Equivalences; 1.5+1.5 Points)

(a) Transform the following formula into CNF by applying the equivalence rules shown in the lecture. For each step, only apply one equivalence rule and also specify it.

$$\phi = (\neg(A \leftrightarrow \neg B) \rightarrow C)$$

(b) Prove that the following formula is a tautology by showing that $\phi \equiv (A \vee \neg A)$ holds. Use the equivalence rules from the lectures, only apply one rule for each step and specify the applied rule.

$$\phi = (A \vee (\neg(A \wedge \neg(\neg A \wedge C)) \vee (A \wedge B)))$$

Exercise 2.4 (Formula Sets; 1.5+1.5 Points)
Consider the following formula set:

$$\text{KB} = \{(A \lor C), (B \lor \neg C), (C \to \neg B)\}$$

(a) Does a model $\mathcal{I}$ of KB exist which is also a model for $\phi = (C \lor \neg A)$? Prove your statement.

(b) Prove that all models $\mathcal{I}$ of KB are also models of $\phi = (A \lor B)$.