Theory of Computer Science

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Exercise Sheet 3

Due: Wednesday, March 21, 2018

Exercise 3.1 (Inference; 1+1+1 Points + 1 Bonus Point)

You'll find a Java program on the lecture website that checks proofs formulated in propositional logic. Use this program to prove the following statements. For a statement of the form WB $\models \varphi$ write a text file containing a derivation that only uses formulas from WB as assumptions and that has φ in its last line. An example for this is contained in the file `proof.txt`.
The program checks WB $\vdash \varphi$. Since the proof system used by the program is correct, this implies WB $\models \varphi$.

(a) $\{A, B\} \models ((A \land B) \lor C)$

(b) $\{((A \lor B) \to (A \to C)), A\} \models C$

(c) $\{((C \lor D) \leftrightarrow (A \land B)), \neg E, (((A \land B) \land (C \lor D)) \to E)\} \models \neg(A \land B)$
For this exercise, extend the calculus by a new rule *negation-introduction*:

$$\frac{(\varphi \to \psi), (\varphi \to \neg\psi)}{\neg\varphi}$$

(d) *Bonus exercise:* To show that a calculus is correct, we have to prove that all rules are correct. Show the correctness of the rule *negation-introduction*

Note on the submission process: please create one text file for each exercise part which contains the derivation. The program must be able to parse the file and accept the derivation as correct. The new rule (*negation-introduction*) requires a new line in the program. Copy this line on your regular submission. The bonus exercise cannot be solved with the program.

Exercise 3.2 (Resolution Calculus; 2 Points)

Consider the following knowledge base

KB $= \{(A \leftrightarrow \neg D), (\neg A \to (B \lor C)), ((A \to E) \land (B \lor C \lor F)), (E \to (F \to (B \lor C))), (C \to G), (G \to \neg C)\}$.

Use the resolution calculus to show that KB $\models (B \land \neg C)$.
Note: A proof using resolution consists of three steps (see lecture slides for an example). Use the notation from the lecture slides in particular in the last step, that is, use one line for each derived clause together with the derivation's justification. Schöning also uses this notation for the third step on page 36 (the first part of the example, not the visualization).

Exercise 3.3 (Syntax of Predicate Logic; 3 Points)

Mark in the following expressions and in all partial expressions all contained syntactic constructions and specify special cases. First-order formulas are interpreted in the context of the following signature: $\mathcal{S} = \langle \mathcal{V}, \mathcal{K}, \mathcal{F}, \mathcal{P} \rangle$ mit $\mathcal{V} = \{x, y, z\}$, $\mathcal{K} = \{\mathrm{c}, \mathrm{d}\}$, $\mathcal{F} = \{\mathrm{f}, \mathrm{g}, \mathrm{h}, \mathrm{j}, \mathrm{k}, \mathrm{l}\}$, $\mathcal{P} = \{\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$. Is it ...:

- a *term* (if so, is it a *base term*)?
- a *formula* (if so, is it a *sentence*)?
- a *statement*?
- a *variable symbol* (if so, is the variable free or bound, by which quantifier)?
- a *constant symbol*?
- a *function symbol* (with which arity)?
- a *predicate symbol* (with which arity)?
- a *function*?
- a *set*?
- a *object in the universe*?
- a syntactic mistake?

(a) $\mathrm{f}(x, y, z)$

(b) $\mathrm{Q}(x, y)$

(c) $\mathcal{I}, \alpha \models (\mathrm{R}(x) \wedge (\mathrm{g}(x) = \mathrm{c}))$

(d) $(\mathrm{R}(x) \wedge \forall x\, \mathrm{S}(x))$

(e) $\forall x\, (\mathrm{g}(x) = \mathrm{h}(x)) \equiv (\mathrm{j}(x) = \mathrm{k}(x))$

(f) $\forall x \exists y\, ((\mathrm{l}(x, \mathrm{g}(x)) = y) \wedge \mathrm{P}(x, y, z))$

(g) $\forall x\, (\mathrm{R}(x) \leftrightarrow \mathrm{Q}(x, \mathrm{h}(\mathrm{d})))$

(h) $\mathrm{f}^{\mathcal{I},\alpha}(x^{\mathcal{I},\alpha}, y^{\mathcal{I},\alpha}, z^{\mathcal{I},\alpha})$

Exercise 3.4 (Predicate Logic; 2 Points)

Consider the following predicate logic formula φ with the signature $\langle \{x\}, \{\mathrm{c}\}, \{\mathrm{f}\}, \{\mathrm{P}\} \rangle$.

$$\varphi = (\exists x\, (\mathrm{P}(x) \wedge \neg \mathrm{P}(\mathrm{f}(x))) \wedge \forall x\, \neg(\mathrm{f}(x) = \mathrm{c}))$$

Specify a model $\mathcal{I}$ of φ with $\mathcal{I} = \left\langle U, \cdot^{\mathcal{I}} \right\rangle$ and $U = \{u_1, u_2, u_3\}$. Prove that $\mathcal{I} \models \varphi$. Why is no variable assignment α required to specify a model of φ?