Foundations of Artificial Intelligence

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Exercise Sheet 8
Due: April 24, 2019

Due to the Easter break, this sheet only has a total of 8 marks.

Exercise 8.1 (2+1 marks)

Consider the constraint network that is given by the graph coloring problem of a graph $G = \langle V, E \rangle$. The set of vertices V contains a vertex for each Swiss canton, and E is such that two vertices v and v' are connected iff the cantons v and v' share a border. A description of G can be downloaded from the website of the course.

(a) Provide a cutset $V' \subseteq V$ for G that is as small as possible (it is not necessary to provide an explanation how you have found V'). As a reminder, a cutset of a graph is defined as a set of vertices that is such that the induced subgraph that is obtained by removing these vertices results in an acyclic graph.

   Note: You get 2 marks for your solution if your cutset is optimal, 1 mark if your cutset contains exactly one more vertex than an optimal cutset and 0 marks otherwise.

(b) Assume we are interested in coloring G with 4 colors. Provide a worst-case runtime estimate of the algorithm based on cutset conditioning if your cutset from the first part of this exercise is used (i.e., compute an upper bound for the number of considered assignments). Compare your result to the estimated runtime if no cutset is used.

Hint: It is simpler to solve this exercise if a graph visualization tool like, for instance, *graphviz* is used. The description of G that can be found on the website of the course is formatted for graphviz. If you use Linux, you can create a pdf visualization of the graph with:

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dot -T pdf -o cantons.pdf cantons.dot
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Exercise 8.2 (0.5+0.5+0.5+0.5 marks)

Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions that clearly express what they encode.

(a) If Alice drinks lemonade or water, she is not thirsty.

(b) If Alice is thirsty and she has money, she buys lemonade.

(c) Either Alice drinks lemonade or she eats a sandwich (but not both).

(d) Alice only drinks lemonade in summer.

Exercise 8.3 (3 marks)

Consider the following syntax extension for propositional logic: $\psi \leftrightarrow \eta$ is a propositional formula if ψ and η are propositional formulas. Furthermore, let the semantics of $\leftrightarrow$ be defined by

$$I \models \psi \leftrightarrow \eta \text{ iff } I \models \psi \rightarrow \eta \text{ and } I \models \eta \rightarrow \psi.$$

Now consider the propositional formula

$$\varphi = ((A \rightarrow B) \vee C) \wedge (D \leftrightarrow \neg(C \rightarrow A))$$

and the interpretation

$$I = \{A \mapsto \mathbf{T}, B \mapsto \mathbf{T}, C \mapsto \mathbf{F}, D \mapsto \mathbf{F}\}.$$

Use only the definition of $\models$ to show that $I \models \varphi$. Provide each intermediate step.

Important: Solutions should be submitted in groups of two students. However, only one student should upload the solution. Please provide both student names on each file and each page you submit. We can only accept a single PDF or a ZIP file containing *.java or *.pddl files and a single PDF.