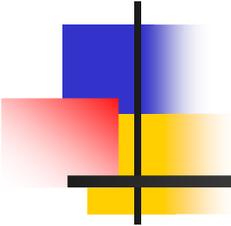


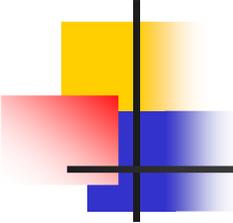
On Transposition Tables for Single-Agent Search and Planning: Summary of Results (Akagi, Kishimoto, Fukunaga) [1]



Seminar Search and Optimization

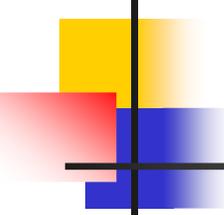
Basel, 23.10.2012

Christian Mächler



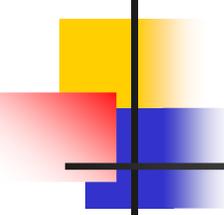
Overview

- Review of A* and IDA*
- Transposition Tables
- IDA* + TT
 - Case studies
 - Experimental results
- Questions



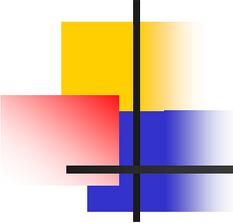
A* search algorithm

- Best-first search
- Finds the least cost path with an admissible heuristic
- Path-cost function $g(x)$
- Heuristic estimate $h(x)$
- Next node: lowest $f(x) = g(x) + h(x)$
- Open set: Priority queue of nodes to be traversed
- Closed set: Nodes already visited



Iterative Deepening A* (IDA*)

- Iterative deepening depth-first search
- Uses the f-costs as next limit/bound
- Costs exceed limit → path cut off
- The limit for the next iteration is set to the lowest-cost node that was pruned during the previous iteration
- Finds an optimal solution if an admissible heuristic is used



IDA* Pseudocode

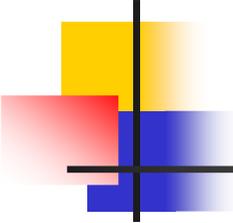
```
function DFS(n, bound, path): real  
  1: if n is a goal state then  
  2:   solved := true; answer := path; return (0);  
  3: end if  
  4: if successors(n) =  $\emptyset$  then  
  5:   new_bound :=  $\infty$  ;  
  6: else  
  7:   new_bound :=  $\min\{\text{BD}(m) \mid m \in \text{successors}(n)\}$ ;  
  8: end if  
  9: return (new_bound);
```

where $\text{BD}(m) :=$

Case 1: ∞ , if *path* + *m* forms a cycle

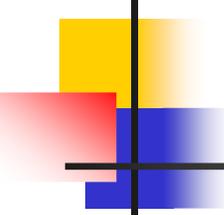
Case 2: $c(n, m) + \text{DFS}(m, \text{bound} - c(n, m), \text{path} + m)$,
if $c(n, m) + h(m) \leq \text{bound}$

Case 3: $c(n, m) + h(m)$, if $c(n, m) + h(m) > \text{bound}$



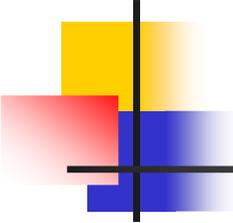
A* versus IDA*

	A*	IDA*
Memory requirements	large	small
Re-expansion of duplicate states reached via different paths	no (with a consistent heuristic)	yes



Consistent heuristic

- $h(G) = 0$
- $h(N) \leq c(N,P) + h(P)$
- P: successor of N
- $c(N,P)$: costs from N to P



Transposition Table (TT)

- Hash table of positions/nodes analyzed so far up to a certain depth
- Used to avoid re-expanding the same node (or re-evaluating the same position)
- Usually not enough memory → a replacement strategy/policy has to be used
- For IDA* the TT is a cache where the keys are states and the entries contain the estimated cost to a solution state

DFSTT1 a straightforward extension of DFS

function DFSTT1($n, bound, path$): **real**

```
1: if  $n$  is a goal state then  
2:    $solved := true; answer := path; \mathbf{return} (0);$   
3: end if  
4: if  $successors(n) = \emptyset$  then  
5:    $new\_bound := \infty;$   
6: else  
7:    $new\_bound := \min\{BD(m) | m \in successors(n)\};$   
8: end if  
9: store  $(n, new\_bound)$  in  $TT$ ;  
10: return  $(new\_bound)$ ;
```

where $BD(m) :=$

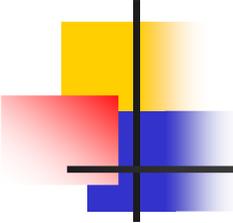
Case 1: ∞ , if $path + m$ forms a cycle

Case 2: $c(n, m) + DFSTT1(m, bound - c(n, m), path + m)$,
if $c(n, m) + Lookup(m) \leq bound$

Case 3: $c(n, m) + Lookup(m)$,
if $c(n, m) + Lookup(m) > bound$

function Lookup(m, TT): **real**

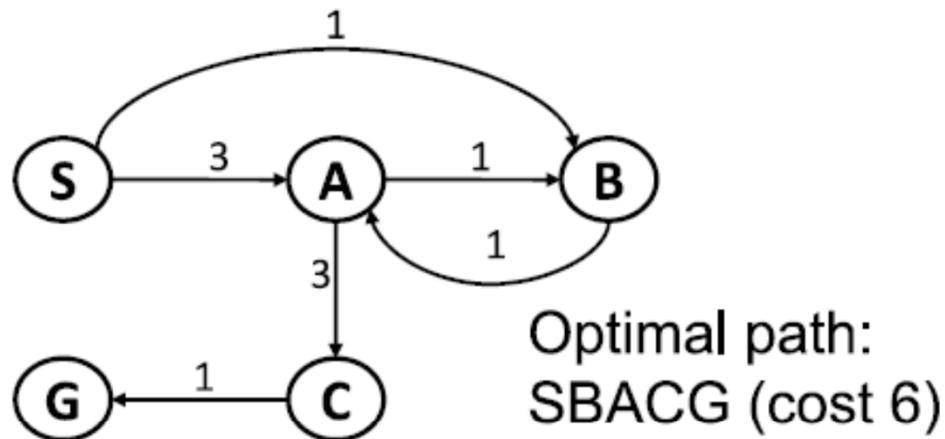
```
1: if  $m$  is in  $TT$  then  
2:   return  $esti(m)$ ;  
3: else  
4:   store  $(m, h(m))$  in  $TT$   
5:   return  $h(m)$   
6: end if
```



Properties of DFSTT1

- Given a consistent heuristic, IDA* using DFSTT1 with an infinite capacity \mathbb{T} is admissible
- Given a consistent heuristic, IDA* using DFSTT1 with a finite-capacity \mathbb{T} is not admissible (for some replacement policies)
- Given an admissible inconsistent heuristic, IDA* using DFSTT1 is not admissible

Counterexample



$$c(S, A) = c(A, C) = 3$$

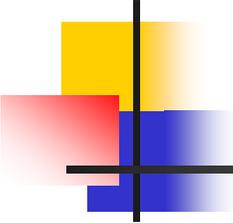
$$c(S, B) = c(A, B) = c(B, A) = c(C, G) = 1$$

$$h(S) = 2, h(A) = 2, h(B) = h(C) = 1$$

Successor ordering for S: A, B

Successor ordering for A: B, C

[1]



DFSTT2

function DFSTT2($n, bound, path$): (real,real)

```
1: if  $n$  is a goal state then  
2:    $solved := true; answer := path; \text{return } (0, 0);$   
3: end if  
4: if  $successors(n) = \emptyset$  then  
5:    $new\_esti := \infty; new\_bound := \infty;$   
6: else  
7:    $new\_esti := \min\{ET(m) | m \in successors(n)\};$   
8:    $new\_bound := \min\{BD(m) | m \in successors(n)\};$   
9: end if  
10:  $\text{store } (n, new\_esti)$  in  $TT$ ;  
11: return ( $new\_esti, new\_bound$ );
```

Where $ET(m) :=$

Case 1: $c(n, m) + \text{Lookup}(m)$, if $path + m$ forms a cycle

Case 2: $c(n, m) + \text{DFSTT2}(m, bound - c(n, m), path + m)[0]$,
if $c(n, m) + \text{Lookup}(m) \leq bound$

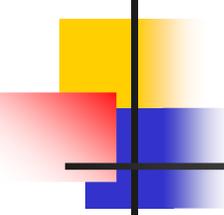
Case 3: $c(n, m) + \text{Lookup}(m)$,
if $c(n, m) + \text{Lookup}(m) > bound$

$BD(m) :=$

Case 1: ∞ , if $path + m$ forms a cycle

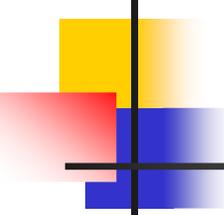
Case 2: $c(n, m) + \text{DFSTT2}(m, bound - c(n, m), path + m)[1]$,
if $c(n, m) + \text{Lookup}(m) \leq bound$

Case 3: $c(n, m) + \text{Lookup}(m)$,
if $c(n, m) + \text{Lookup}(m) > bound$



Properties of DFSTT2

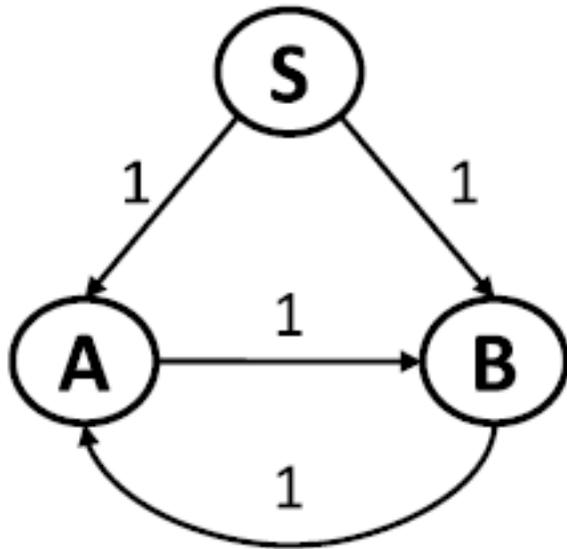
- Given an admissible heuristic function $IDA^* + DFSTT2$ is admissible
- But $IDA^* + DFSTT2$ is not complete



RollingStone strategy (RS)

- Stores $bound - g(n) + 1$ before searching the subtree
- $g(n)$: costs to reach node n
- Cycling back into this state will cause a cutoff because $g(s)$ will be higher than its previous value
→ no cycle detection needed
- Admissible

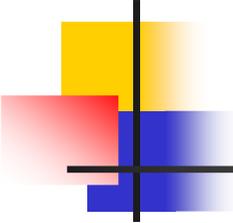
Counterexample



$$c(S, A) = c(S, B) = 1$$

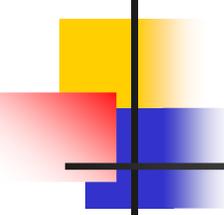
$$h(S) = h(B) = 2, h(A) = 1$$

No solution



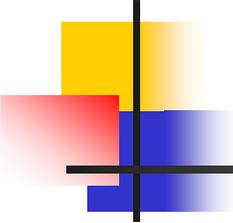
DFSTT3

- Not only store esti, but also the associated g-cost
- Allows to determine if a revisited node was already reached via a shorter path
- This allows to label dead ends
- Complete
- But large performance degradation



Replacement Policies

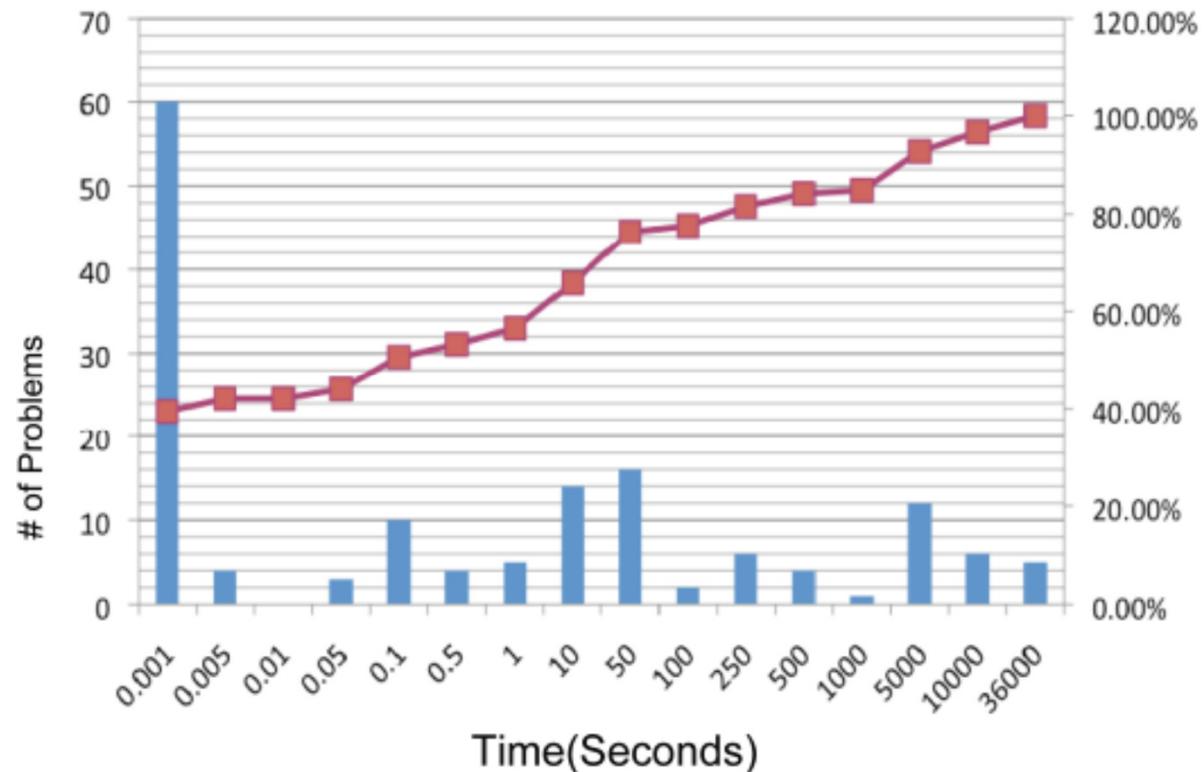
- No replacement
 - Stochastic Node Caching
 - Collision-based replacement
 - Batch replacement
- Sort criterias:
- subtree size
 - backed up cost estimate
 - # accesses



Experimental results

Algorithm	TT Replacement Policy	Num Solved	Tot. Runtime (seconds)
A*		173	539
DFS	No TT	128	178098
DFSTT2	TT, No Replace	183	73477
DFSTT2	Replace 0.3, subtree size	194	52256
RS	Replace 0.3, subtree size	195	68319
DFSTT2+RS	TT, No Replace	189	106147
DFSTT2+RS	Stochastic Caching, $p=0.001$	187	213765
DFSTT2+RS	Replace 0.3, est	194	40290
DFSTT2+RS	Replace 0.3, subtree size	195	66960
DFSTT2+RS	Replace 0.3, access freq.	194	39187
DFSTT2+RS	Collision, est	189	141249
DFSTT2+RS	Collision, subtree size	192	114057
DFSTT3+RS	Replace 0.3, subtree size	152	170296

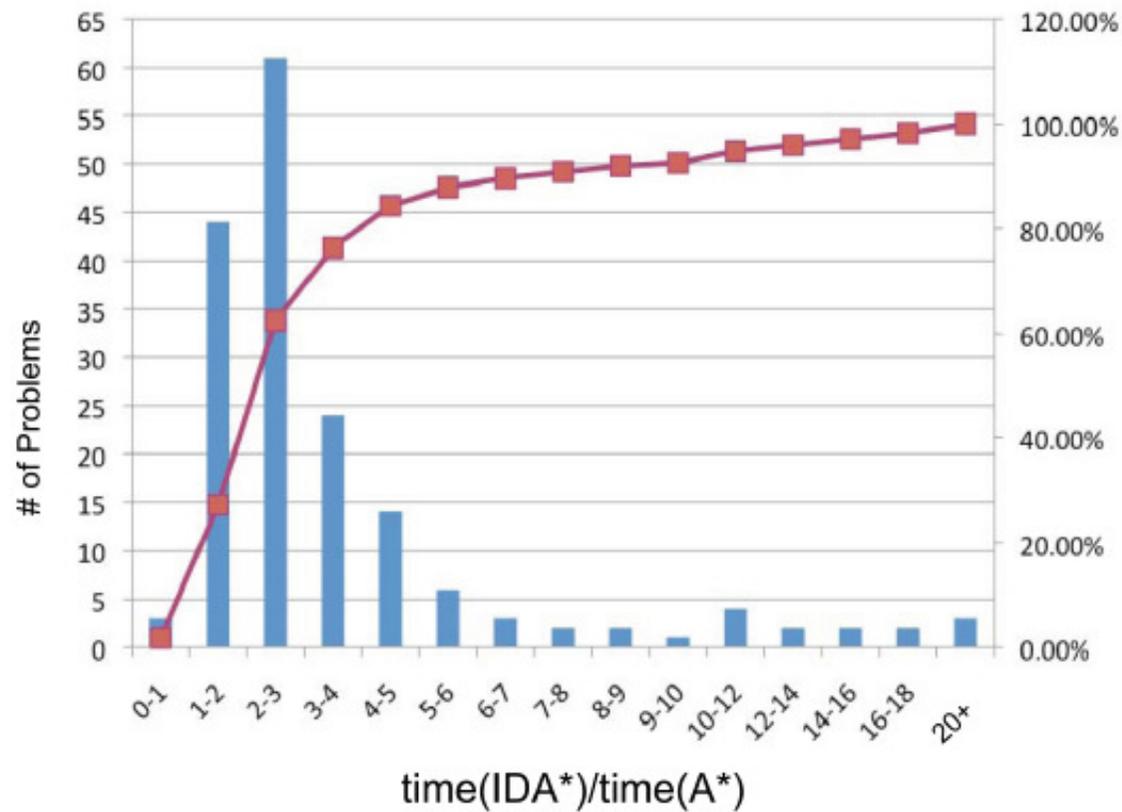
Experimental results



Runtime distribution of IDA* + DFSTT2

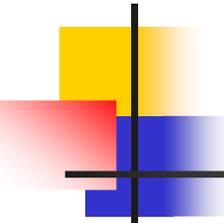
[1]

Experimental results



IDA* + DFSTT2 + RS vs. A* performance

[1]



End of presentation

- Questions?