A Beginner's Introduction to Heuristic Search Planning

6. Delete Relaxation and Landmarks

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Overview

Reminder

Recall from Part 4:

Delete Relaxation

Estimate cost to goal by considering simpler planning task without negative side effects of actions.

Landmarks

An action set A is a landmark if all plans include an action from A.

Compute a set of landmarks and use it to derive a cost estimate (e.g., by counting the number of landmarks).

In This Part

Now:

- principled way of deriving landmarks for delete relaxation
- principled ways of exploiting landmarks for heuristics

⇝ basis of landmark-cut heuristic (Helmert & Domshlak, ICAPS 2009) and improved LM-Cut (Bonet & Helmert, ECAI 2010)

Relaxed Planning

Delete Relaxation

delete relaxation: ignore "bad effects" of actions

- What is a bad effect?
- easy for STRIPS: it's always "better for us" if a fact is true!

$\rightsquigarrow$ bad effect = delete effect

$\rightsquigarrow$ delete relaxation of a task: drop all delete effects

Use delete relaxation as basis for heuristics:

- in each state, estimate cost to the goal in delete relaxation

Relaxed Planning Tasks

It is convenient to use special-purpose notation for relaxed tasks:

Relaxed Planning Task

F: finite set of facts

- initial facts $I \subseteq F$ are given
- goal facts $G \subseteq F$ must be reached
- operators of the form $o[4] : a, b \rightarrow c, d$

  read: If we already have facts a and b (preconditions $pre(o)$),
  we can apply o, paying 4 units (cost $cost(o)$),
  to obtain facts c and d (effects $eff(o)$)

For simplicity: assume $I = \{i\}$, $G = \{g\}$, all $pre(o) \neq \emptyset$

Example: Relaxed Planning Task

Example

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
$o_3[5] : i \rightarrow b, c$
$o_4[0] : a, b, c \rightarrow g$

Example: Relaxed Planning Task

Example

$o_1[3] : i \rightarrow a, b$
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One way to reach $\{g\}$ from $\{i\}$:

- apply sequence o_1, o_2, o_4 (plan)
- cost: $3 + 4 + 0 = 7$ (optimal)

Optimal Relaxed Cost

- $h^+(I)$: minimal total cost to reach G from I
- NP-hard to compute (Bylander, AIJ 1994)
  or approximate by constant factor (Betz & Helmert, KI 2009)

$\rightsquigarrow$ use polynomial-time admissible heuristics

Landmarks

Landmarks

The most accurate current heuristics are based on landmarks.

Definition (Landmark)

A (disjunctive action) landmark is a set of operators L such that each plan must contain some element of L.

The cost of a landmark, $cost(L)$, is $\min_{o \in L} cost(o)$.

$\rightsquigarrow$ the cost of any landmark is a (crude) admissible heuristic

Example: Landmarks

Example

$o_1[3] : i \rightarrow a, b$

$o_2[4] : i \rightarrow a, c$

$o_3[5] : i \rightarrow b, c$

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Example: Landmarks

Example

$o_1[3] : i \to a, b$
$o_2[4] : i \to a, c$
$o_3[5] : i \to b, c$
$o_4[0] : a, b, c \to g$

Some landmarks:

- $W = \{o_4\}$ (cost 0)
- $X = \{o_1, o_2\}$ (cost 3)
- $Y = \{o_1, o_3\}$ (cost 3)
- $Z = \{o_2, o_3\}$ (cost 4)
- but also: $\{o_1, o_2, o_3\}$ (cost 3), $\{o_1, o_2, o_4\}$ (cost 0), ...

Exploiting Landmarks

Exploiting Landmarks

Assume we are given landmark set $\mathcal{L} = \{W, X, Y, Z\}$.
(later: how to find such landmarks)

How do we exploit $\mathcal{L}$ for heuristics?

- sum of costs $0 + 3 + 3 + 4 = 10 \rightsquigarrow$ inadmissible!
- maximum of costs: $\max\{0, 3, 3, 4\} = 4 \rightsquigarrow$ weak
- best previous approach: optimal cost partitioning

Landmark Heuristics with Optimal Cost Partitioning

Optimal Cost Partitioning (Karpas & Domshlak, IJCAI 2009)

Idea: Derive a linear program (LP) from $\mathcal{L}$.

- one variable per landmark
- one constraint per operator

h^{L} value: objective value of the LP

Example: Optimal Cost Partitioning

Example

$cost(o_1) = 3, \quad cost(o_2) = 4, \quad cost(o_3) = 5, \quad cost(o_4) = 0$

$\mathcal{L} = \{W, X, Y, Z\}$
with $W = \{o_4\}, \quad X = \{o_1, o_2\}, \quad Y = \{o_1, o_3\}, \quad Z = \{o_2, o_3\}$

LP: maximize $w + x + y + z$ subject to $w, x, y, z \geq 0$ and

$$\begin{array}{lllllllll} & x & + & y & & & \leq & 3 \\ & x & + & & & z & \leq & 4 \\ & & & y & + & z & \leq & 5 \\ w & & & & & & \leq & 0 \end{array}$$

Example: Optimal Cost Partitioning

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$$
\begin{array}{ccccccccccc}
 & & x & + & y & & & \leq & 3 & & o_1 \\
 & & x & + & & & z & \leq & 4 & & o_2 \\
 & & & & y & + & z & \leq & 5 & & o_3 \\
w & & & & & & & \leq & 0 & & o_4 \\
W & & X & & Y & & Z & & & &
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$$

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 & & x & + & & & z & \leq & 4 & o_2 \\
 & & & & y & + & z & \leq & 5 & o_3 \\
w & & & & & & & \leq & 0 & o_4 \\
W & & X & & Y & & Z & & &
\end{array}$$

solution: $w = 0, \; x = 1, \; y = 2, \; z = 3 \quad \rightsquigarrow \quad h^{\mathrm{L}}(I) = 6$

Beyond Optimal Cost Partitioning

- $h^{\mathsf{L}}(I) = 6$ is a good estimate, but $h^{+}(I) = 7$!
- Can we do better with the same information?

Hitting Sets

Definition (Hitting Set)

Given: finite set A, subset family $\mathcal{F} \subseteq 2^A$, costs $c : A \to \mathbb{R}_0^+$

Hitting set:

- subset $H \subseteq A$ that "hits" all subsets in $\mathcal{F}$:
  $H \cap S \neq \emptyset$ for all $S \in \mathcal{F}$
- cost of H: $\sum_{a \in H} c(a)$

Minimum hitting set (MHS):

- minimizes cost
- classical NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$A = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{W, X, Y, Z\}$
with $W = \{o_4\}$, $X = \{o_1, o_2\}$, $Y = \{o_1, o_3\}$, $Z = \{o_2, o_3\}$

$c(o_1) = 3$, $c(o_2) = 4$, $c(o_3) = 5$, $c(o_4) = 0$

Minimum hitting set:

Example: Hitting Sets

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with $W = \{o_4\}$, $X = \{o_1, o_2\}$, $Y = \{o_1, o_3\}$, $Z = \{o_2, o_3\}$

$c(o_1) = 3$, $c(o_2) = 4$, $c(o_3) = 5$, $c(o_4) = 0$

Minimum hitting set: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Landmarks

- can view landmark sets (with operator costs) as instances of minimum hitting set problem
- here, we got an admissible estimate that dominated h^{L}
- coincidence?

Hitting Set Heuristics

Let $\mathcal{L}$ be a set of landmarks.

Theorem (Hitting Set Heuristics are Admissible)

Let $h^{\mathsf{MHS}}(I)$ be the minimum hitting set cost for $\langle O, \mathcal{L}, cost \rangle$. Then:

1. $h^{\mathsf{MHS}}(I) \leq h^{+}(I)$ *(hitting set heuristics are admissible)*
2. $h^{\mathsf{MHS}}(I) \geq h^{\mathsf{L}}(I)$ *(hitting sets dominate cost partitioning)*

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Proof sketch:

1. plans are hitting sets (by definition of landmarks)
2. cost partitioning LP is dual of LP relaxation of hitting set integer program

Generating Landmarks

Generating Landmarks

How do we generate landmarks in the first place?

Justification Graphs

Definition (Precondition Choice Function)

A precondition choice function (pcf) $D : O \to F$ maps each operator to one of its preconditions.

Definition (Justification Graph)

The justification graph for pcf D is an arc-labeled digraph with

- vertices: the facts F
- arcs: arc $D(o) \xrightarrow{o} e$ for each operator o and effect $e \in \mathit{eff}(o)$

Example: Justification Graph

Example

pcf D: $D(o_1) = D(o_2) = D(o_3) = i$, $D(o_4) = b$

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
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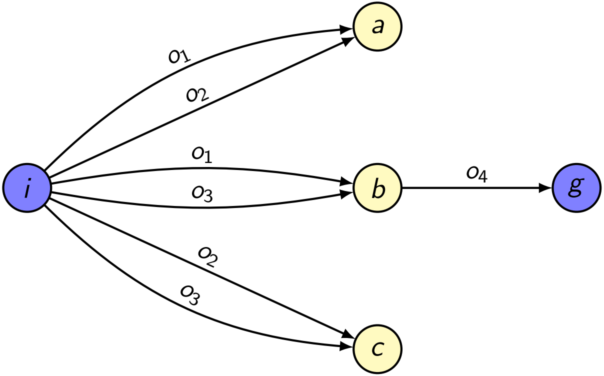

Cuts

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Theorem (Cuts are Landmarks)

Let C be any cut of the justification graph for any pcf. Then the labels of C form a landmark.

Example: Cuts of a Justification Graph

Example

Landmark $W = \{o_4\}$ (cost 0)

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
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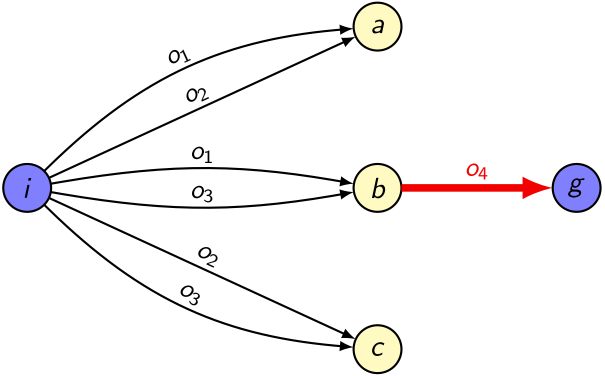

Example: Cuts of a Justification Graph

Example

Landmark $X = \{o_1, o_2\}$ (cost 3)

$o_1[3] : i \rightarrow a, b$

$o_2[4] : i \rightarrow a, c$

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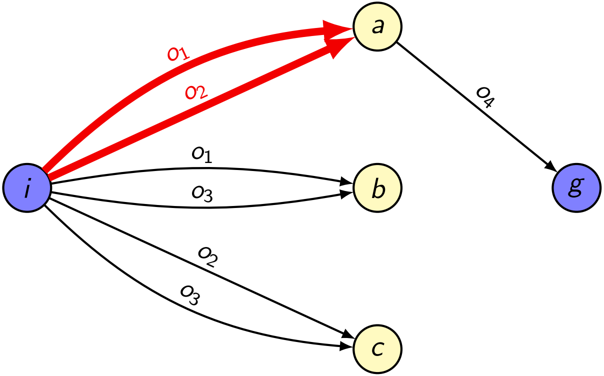

Example: Cuts of a Justification Graph

Example

Landmark $Y = \{o_1, o_3\}$ (cost 3)

$o_1[3] : i \to a, b$
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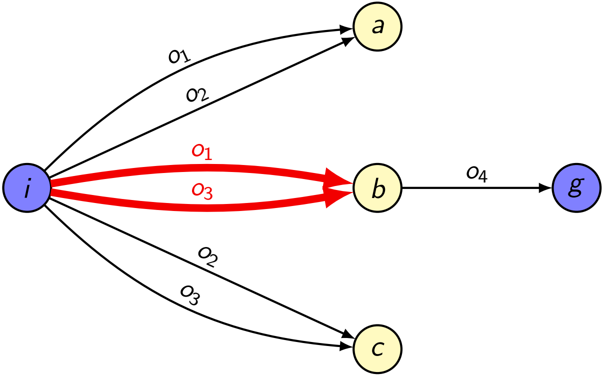

Example: Cuts of a Justification Graph

Example

Landmark $Z = \{o_2, o_3\}$ (cost 4)

$o_1[3] : i \to a, b$
$o_2[4] : i \to a, c$
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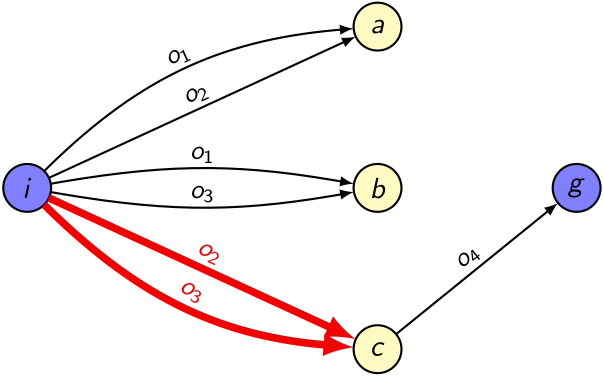
$o_4[0] : a, b, c \to g$



Power of Justification Graph Cuts

- Which landmarks can be generated with the cut method?

Power of Justification Graph Cuts

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- All interesting ones!

Theorem (Perfect Hitting Set Heuristic)

Let $\mathcal{L}$ be the set of all "cut landmarks".
Then $h^{\mathrm{MHS}}(I) = h^{+}(I)$.

$\rightsquigarrow$ hitting set heuristic over $\mathcal{L}$ is perfect

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Proof sketch:

- We show that every hitting set H for $\mathcal{L}$ induces a plan.
- Assume that some hitting set H does not induce a plan.
- We construct a pcf and cut s.t. H does not hit the landmark.
- Contradiction!

Summary

Summary

- Landmarks of delete-relaxed tasks are a good source of heuristic information.
- Optimal cost partitioning is an admissible method for combining information from multiple landmarks.
- Hitting sets for landmarks are more informative than optimal cost partitioning (but NP-hard to compute).
- Cuts in justification graphs offer a principled way of generating landmarks.
- Hitting sets over all cut landmarks are perfect heuristics for delete relaxations.