Overview
Recall from Part 4:

**Delete Relaxation**

Estimate cost to goal by considering simpler planning task **without negative side effects** of actions.

**Landmarks**

An action set $A$ is a **landmark** if all plans include an action from $A$. Compute a set of landmarks and use it to derive a cost estimate (e.g., by counting the number of landmarks).
Now:

- principled way of deriving landmarks for delete relaxation
- principled ways of exploiting landmarks for heuristics
  
  basis of landmark-cut heuristic (Helmert & Domshlak, ICAPS 2009) and improved LM-Cut (Bonet & Helmert, ECAI 2010)
Relaxed Planning
**Delete Relaxation**

**delete relaxation**: ignore “bad effects” of actions

- **What is a bad effect?**
  - easy for STRIPS: it’s always “better for us” if a fact is true!

  \[\Rightarrow \text{bad effect} = \text{delete effect}\]

  \[\Rightarrow \text{delete relaxation} \text{ of a task: drop all delete effects}\]

Use delete relaxation as basis for heuristics:

- in each state, estimate cost to the goal in delete relaxation
It is convenient to use special-purpose notation for relaxed tasks:

**Relaxed Planning Task**

*F*: finite set of facts

- **initial facts** *I* ⊆ *F* are given
- **goal facts** *G* ⊆ *F* must be reached
- **operators** of the form *o*[4]: *a*, *b* → *c*, *d*

read: If we already have facts *a* and *b* (preconditions *pre(o)*), we can apply *o*, paying 4 units (cost *cost(o)*), to obtain facts *c* and *d* (effects *eff(o)*)

For simplicity: assume *I* = \{i\}, *G* = \{g\}, all *pre(o) ≠ ∅*
Example: Relaxed Planning Task

Example

- $o_1[3] : i \rightarrow a, b$
- $o_2[4] : i \rightarrow a, c$
- $o_3[5] : i \rightarrow b, c$
- $o_4[0] : a, b, c \rightarrow g$

One way to reach $\{g\}$ from $\{i\}$:
Apply sequence $o_1, o_2, o_4$ (plan)
Cost: $3 + 4 + 0 = 7$ (optimal)
Example: Relaxed Planning Task

Example

\[ o_1[3] : i \rightarrow a, b \]
\[ o_2[4] : i \rightarrow a, c \]
\[ o_3[5] : i \rightarrow b, c \]
\[ o_4[0] : a, b, c \rightarrow g \]

One way to reach \( \{g\} \) from \( \{i\} \):

- apply sequence \( o_1, o_2, o_4 \) (plan)
- cost: \( 3 + 4 + 0 = 7 \) (optimal)
Optimal Relaxed Cost

- $h^+(l)$: minimal total cost to reach $G$ from $l$
- NP-hard to compute (Bylander, AIJ 1994)
  or approximate by constant factor (Betz & Helmert, KI 2009)
  $\Rightarrow$ use polynomial-time admissible heuristics
Landmarks
The most accurate current heuristics are based on landmarks.

**Definition (Landmark)**

A (disjunctive action) **landmark** is a set of operators $L$ such that each plan must contain some element of $L$.

The **cost** of a landmark, $cost(L)$, is $\min_{o \in L} cost(o)$.

⇒ the cost of any landmark is a (crude) admissible heuristic.
Example: Landmarks

Example

\[ o_1[3] : i \rightarrow a, b \]
\[ o_2[4] : i \rightarrow a, c \]
\[ o_3[5] : i \rightarrow b, c \]
\[ o_4[0] : a, b, c \rightarrow g \]
Example: Landmarks

Some landmarks:

- \( W = \{ o_4 \} \) (cost 0)
- \( X = \{ o_1, o_2 \} \) (cost 3)
- \( Y = \{ o_1, o_3 \} \) (cost 3)
- \( Z = \{ o_2, o_3 \} \) (cost 4)
- but also: \( \{ o_1, o_2, o_3 \} \) (cost 3), \( \{ o_1, o_2, o_4 \} \) (cost 0), ...
Exploiting Landmarks
Exploiting Landmarks

Assume we are given landmark set $\mathcal{L} = \{W, X, Y, Z\}$. 
(later: how to find such landmarks)

How do we exploit $\mathcal{L}$ for heuristics?

- sum of costs $0 + 3 + 3 + 4 = 10 \implies \text{inadmissible!}$
- maximum of costs: $\max\{0, 3, 3, 4\} = 4 \implies \text{weak}$
- best previous approach: optimal cost partitioning
Landmark Heuristics with Optimal Cost Partitioning

**Optimal Cost Partitioning (Karpas & Domshlak, IJCAI 2009)**

**Idea:** Derive a linear program (LP) from $\mathcal{L}$.
- one variable per landmark
- one constraint per operator

$h^L$ value: objective value of the LP
Example: Optimal Cost Partitioning

Example

cost(o_1) = 3,  cost(o_2) = 4,  cost(o_3) = 5,  cost(o_4) = 0

\[ \mathcal{L} = \{W, X, Y, Z\} \]

with \( W = \{o_4\} \), \( X = \{o_1, o_2\} \), \( Y = \{o_1, o_3\} \), \( Z = \{o_2, o_3\} \)

LP: maximize \( w + x + y + z \) subject to \( w, x, y, z \geq 0 \) and

\[
\begin{align*}
x + y & \leq 3 \\
x + z & \leq 4 \\
y + z & \leq 5 \\
w & \leq 0
\end{align*}
\]
### Example: Optimal Cost Partitioning

**Example**

\[ \begin{align*}
  \text{cost}(o_1) &= 3, \quad \text{cost}(o_2) = 4, \quad \text{cost}(o_3) = 5, \quad \text{cost}(o_4) = 0 \\
  \mathcal{L} &= \{W, X, Y, Z\} \\
  \text{with } W &= \{o_4\}, \quad X = \{o_1, o_2\}, \quad Y = \{o_1, o_3\}, \quad Z = \{o_2, o_3\}
\end{align*} \]

LP: maximize \( w + x + y + z \) subject to \( w, x, y, z \geq 0 \) and

\[
\begin{align*}
  x + y & \leq 3 \quad o_1 \\
  x + z & \leq 4 \quad o_2 \\
  y + z & \leq 5 \quad o_3 \\
  w & \leq 0 \quad o_4 \\
  W & \quad X & \quad Y & \quad Z
\end{align*}
\]
Example: Optimal Cost Partitioning

Example

\[ \text{cost}(o_1) = 3, \quad \text{cost}(o_2) = 4, \quad \text{cost}(o_3) = 5, \quad \text{cost}(o_4) = 0 \]

\[ \mathcal{L} = \{W, X, Y, Z\} \]

with \( W = \{o_4\}, \quad X = \{o_1, o_2\}, \quad Y = \{o_1, o_3\}, \quad Z = \{o_2, o_3\} \)

LP: maximize \( w + x + y + z \) subject to \( w, x, y, z \geq 0 \) and

\[
\begin{align*}
    x + y & \leq 3 & o_1 \\
    x + z & \leq 4 & o_2 \\
    y + z & \leq 5 & o_3 \\
    w & \leq 0 & o_4 \\
\end{align*}
\]

\[ W \quad X \quad Y \quad Z \]

solution: \( w = 0, \quad x = 1, \quad y = 2, \quad z = 3 \quad \Rightarrow \quad h^L(I) = 6 \)
Beyond Optimal Cost Partitioning

- $h^{-1}(I) = 6$ is a good estimate, but $h^+(I) = 7$!
- Can we do better with the same information?
**Definition (Hitting Set)**

Given: finite set $A$, subset family $\mathcal{F} \subseteq 2^A$, costs $c : A \rightarrow \mathbb{R}_0^+$

**Hitting set:**
- subset $H \subseteq A$ that “hits” all subsets in $\mathcal{F}$: $H \cap S \neq \emptyset$ for all $S \in \mathcal{F}$
- cost of $H$: $\sum_{a \in H} c(a)$

**Minimum hitting set (MHS):**
- minimizes cost
- classical NP-complete problem (Karp, 1972)
Example: Hitting Sets

Example

\[ A = \{o_1, o_2, o_3, o_4\} \]
\[ \mathcal{F} = \{W, X, Y, Z\} \]
with \[ W = \{o_4\}, \ X = \{o_1, o_2\}, \ Y = \{o_1, o_3\}, \ Z = \{o_2, o_3\} \]
\[ c(o_1) = 3, \ c(o_2) = 4, \ c(o_3) = 5, \ c(o_4) = 0 \]

Minimum hitting set:
Example: Hitting Sets

Example

\[ A = \{o_1, o_2, o_3, o_4\} \]
\[ \mathcal{F} = \{W, X, Y, Z\} \]
with \( W = \{o_4\}, \ X = \{o_1, o_2\}, \ Y = \{o_1, o_3\}, \ Z = \{o_2, o_3\} \)
\[ c(o_1) = 3, \ c(o_2) = 4, \ c(o_3) = 5, \ c(o_4) = 0 \]

Minimum hitting set: \( \{o_1, o_2, o_4\} \) with cost \( 3 + 4 + 0 = 7 \)
Hitting Sets for Landmarks

- can view **landmark sets** (with operator costs) as instances of **minimum hitting set** problem
- here, we got an admissible estimate that dominated $h^L$
- coincidence?
Hitting Set Heuristics

Let $\mathcal{L}$ be a set of landmarks.

Theorem (Hitting Set Heuristics are Admissible)

Let $h^{\text{MHS}}(I)$ be the minimum hitting set cost for $\langle O, \mathcal{L}, \text{cost} \rangle$. Then:

1. $h^{\text{MHS}}(I) \leq h^+(I)$ (hitting set heuristics are admissible)
2. $h^{\text{MHS}}(I) \geq h^L(I)$ (hitting sets dominate cost partitioning)
Let $\mathcal{L}$ be a set of landmarks.

**Theorem (Hitting Set Heuristics are Admissible)**

Let $h^{\text{MHS}}(I)$ be the minimum hitting set cost for $\langle O, \mathcal{L}, \text{cost} \rangle$. Then:

1. $h^{\text{MHS}}(I) \leq h^+(I)$ (hitting set heuristics are admissible)
2. $h^{\text{MHS}}(I) \geq h^L(I)$ (hitting sets dominate cost partitioning)

**Proof sketch:**

1. plans are hitting sets (by definition of landmarks)
2. cost partitioning LP is dual of LP relaxation of hitting set integer program
<table>
<thead>
<tr>
<th>Overview</th>
<th>Relaxed Planning</th>
<th>Landmarks</th>
<th>Exploiting Landmarks</th>
<th>Generating Landmarks</th>
<th>Summary</th>
</tr>
</thead>
</table>

Generating Landmarks
How do we generate landmarks in the first place?
Definition (Precondition Choice Function)

A precondition choice function (pcf) $D : O \rightarrow F$ maps each operator to one of its preconditions.

Definition (Justification Graph)

The justification graph for pcf $D$ is an arc-labeled digraph with

- **vertices:** the facts $F$
- **arcs:** arc $D(o) \rightarrow e$ for each operator $o$ and effect $e \in eff(o)$
Example: Justification Graph

pcf $D$: $D(o_1) = D(o_2) = D(o_3) = i$, $D(o_4) = b$

$o_1[3]: i \rightarrow a, b$

$o_2[4]: i \rightarrow a, c$

$o_3[5]: i \rightarrow b, c$

$o_4[0]: a, b, c \rightarrow g$
Cuts

Definition (Cut)

A cut of a justification graph is a subset of its arcs $C$ such that all paths from $i$ to $g$ use some arc in $C$. 
Cuts

Definition (Cut)
A cut of a justification graph is a subset of its arcs $C$ such that all paths from $i$ to $g$ use some arc in $C$.

Theorem (Cuts are Landmarks)
Let $C$ be any cut of the justification graph for any pcf. Then the labels of $C$ form a landmark.
Example: Cuts of a Justification Graph

Landmark $W = \{ o_4 \}$ (cost 0)

Example

- $o_1[3] : i \rightarrow a, b$
- $o_2[4] : i \rightarrow a, c$
- $o_3[5] : i \rightarrow b, c$
- $o_4[0] : a, b, c \rightarrow g$
Example: Cuts of a Justification Graph

Example

Landmark $X = \{o_1, o_2\}$ (cost 3)

- $o_1[3] : i \rightarrow a, b$
- $o_2[4] : i \rightarrow a, c$
- $o_3[5] : i \rightarrow b, c$
- $o_4[0] : a, b, c \rightarrow g$
Example: Cuts of a Justification Graph

Example

Landmark \( Y = \{ o_1, o_3 \} \) (cost 3)

\[ o_1[3] : i \rightarrow a, b \]
\[ o_2[4] : i \rightarrow a, c \]
\[ o_3[5] : i \rightarrow b, c \]
\[ o_4[0] : a, b, c \rightarrow g \]
Example: Cuts of a Justification Graph

Landmark $Z = \{o_2, o_3\}$ (cost 4)

- $o_1[3] : i \rightarrow a, b$
- $o_2[4] : i \rightarrow a, c$
- $o_3[5] : i \rightarrow b, c$
- $o_4[0] : a, b, c \rightarrow g$
Which landmarks can be generated with the cut method?
### Power of Justification Graph Cuts

- Which landmarks can be generated with the cut method?
- **All interesting ones!**

---

**Theorem (Perfect Hitting Set Heuristic)**

Let $\mathcal{L}$ be the set of all “cut landmarks”. Then $h_{\text{MHS}}^+(I) = h^+(I)$.

$\implies$ hitting set heuristic over $\mathcal{L}$ is **perfect**
Power of Justification Graph Cuts

- Which landmarks can be generated with the cut method?
- All interesting ones!

**Theorem (Perfect Hitting Set Heuristic)**

Let $\mathcal{L}$ be the set of all “cut landmarks”. Then $h^{\text{MHS}}(I) = h^+(I)$.

$\Rightarrow$ hitting set heuristic over $\mathcal{L}$ is perfect

Proof sketch:
- We show that every hitting set $H$ for $\mathcal{L}$ induces a plan.
- Assume that some hitting set $H$ does not induce a plan.
- We construct a pcf and cut s.t. $H$ does not hit the landmark.
- Contradiction!
Summary
Summary

- **Landmarks** of delete-relaxed tasks are a good source of heuristic information.
- **Optimal cost partitioning** is an admissible method for combining information from multiple landmarks.
- **Hitting sets** for landmarks are more informative than optimal cost partitioning (but NP-hard to compute).
- **Cuts in justification graphs** offer a principled way of generating landmarks.
- **Hitting sets over all cut landmarks** are perfect heuristics for delete relaxations.