Best-Case and Worst-Case Behavior of Greedy Best-First Search

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Greedy Best-First Search

- Greedy expansion of states with lowest heuristic values, i.e., estimates of shortest distance to goal
- Search effort depends on tie-breaking decisions
- Problematic to compare heuristics for GBFS, e.g., blind heuristic vs. perfect heuristic

Note:

- Most search effort of A^* does not depend on tie-breaking
- Allows comparing the quality of heuristics for A^*

Complexity Results

Decision Problems

GBFSBestCase / GBFSWorstCase:

Input: state space topology $\mathcal{T}$ with state space $\mathcal{S}$ and heuristic h, $k \in \mathbb{N}_0$

Question: Does there exist a GBFS run on $\mathcal{T}$ that expands at most/least K states?

Theorem

GBFSBestCase and GBFSWorstCase are NP-complete in general and polynomial-time computable for undirected state spaces or state space topologies without overlapping craters and benches.

Hardness comes from . . .

- Overlapping benches and craters that are reachable on different paths
- Combinatorial problem

High-Water Mark

Definition

The high-water mark of state s is

$$hwm(s) := \begin{cases} \min_{\rho \in GoalPaths(s)}(\max_{s' \in \rho} h(s')) & \text{if } GoalPaths(s) \neq \emptyset \\ \infty & \text{otherwise} \end{cases}$$

- Path of least resistance
- Highest heuristic value that GBFS expands in a search starting from s

Search Behavior

Definition

The high-water mark of a set of states S is

$$hwm(S) := \min_{s \in S} hwm(s)$$

- GBFS acts globally on states from *Open* list

Theorems

- Surely expands $s \in Open$ with $h(s) < hwm(Open)$ [crater state]
- Possibly expands $s \in Open$ with $h(s) = hwm(Open)$ [surface state] depending on tie-breaking
- Never expands $s \in Open$ with $h(s) > hwm(Open)$
- Makes progress when expanding $s \in Open$ with $hwm(succ(s)) < hwm(Open) = hwm(s) = h(s)$ [progress state]

- Progress is characterized locally
- Sequence of GBFS episodes
- Clear *Open* before each episode

Topological Structures

- Bench with surface and craters
- Bench transition system
- Bench relates to uninformed heuristic region
- Surface relates to plateau
- Crater relates to local minimum
- Bench transition system depicts progress

Best-Case and Worst-Case Behavior

Best case:

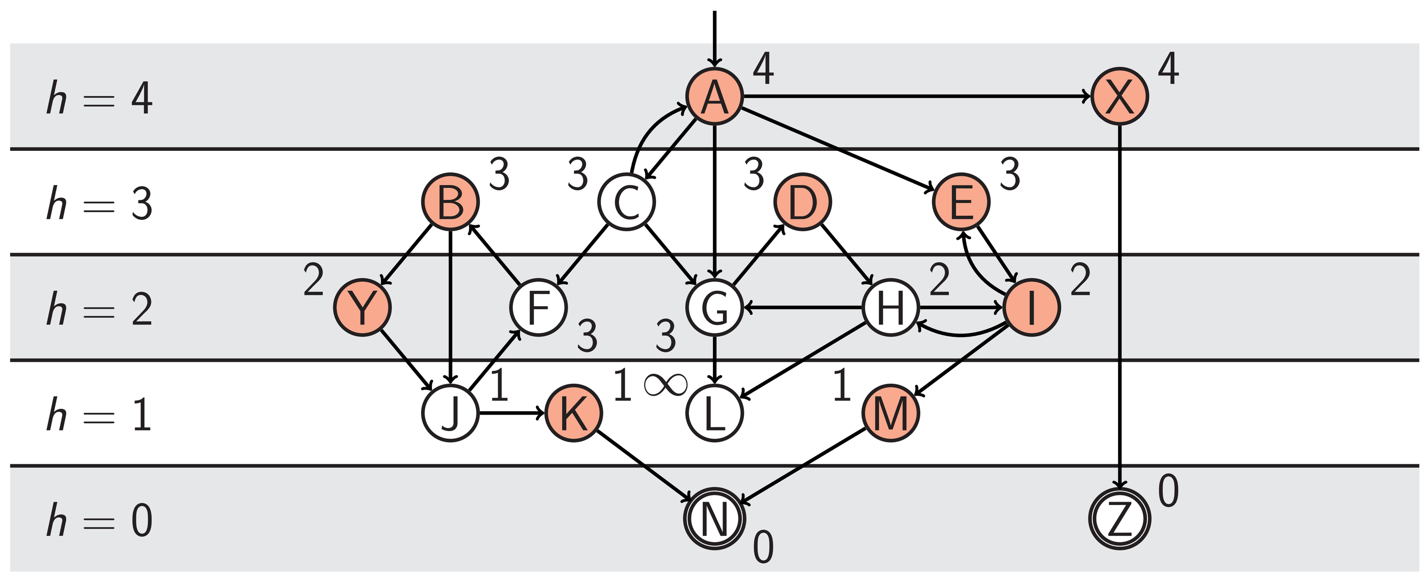
- State path that minimizes the number of expanded states
- Counts crater states that are necessarily expanded along the path

Worst case:

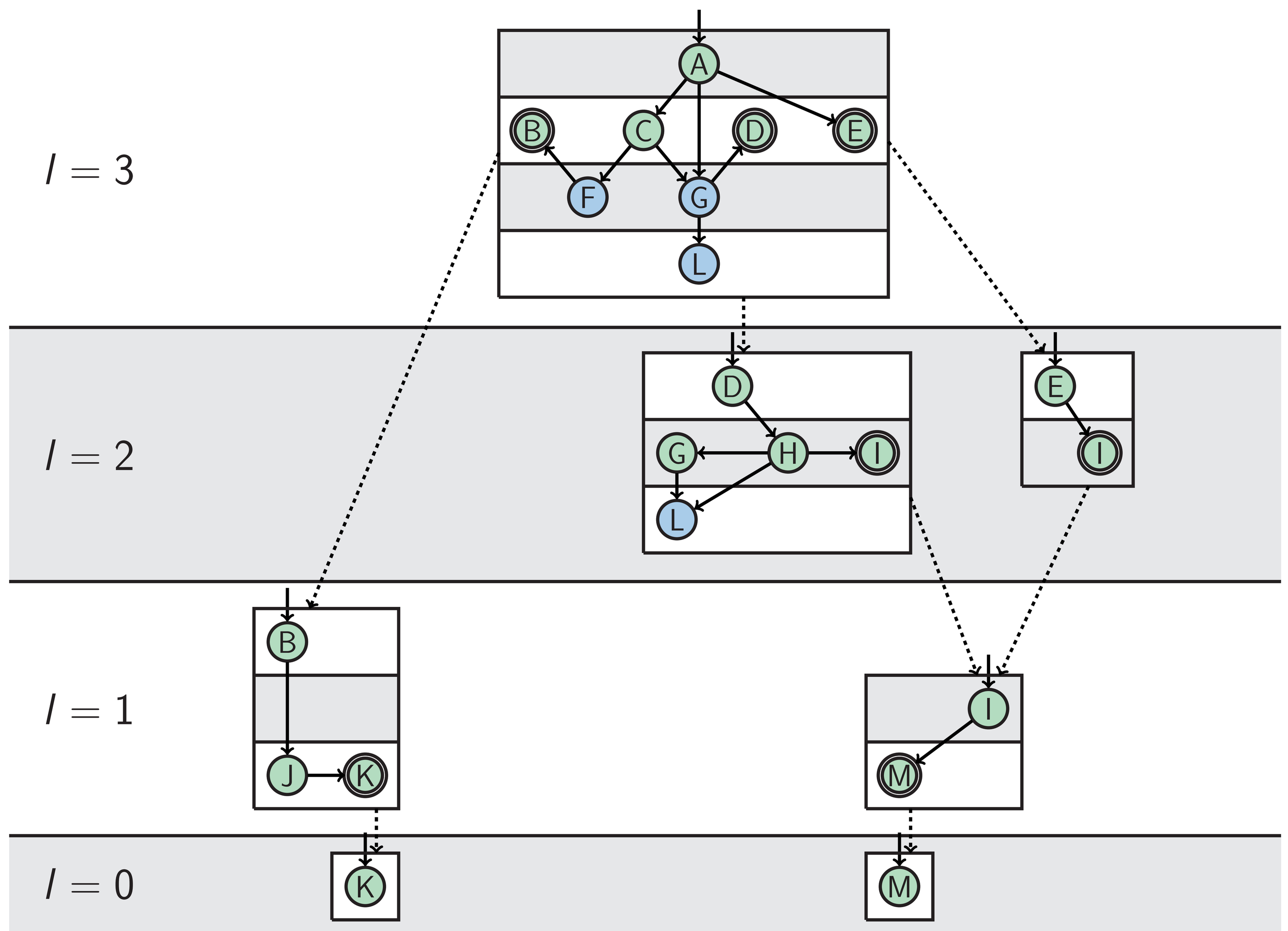
- Bench path that maximizes the number of expanded states
- Counts all non-progress states of benches along the path

Example

State space topology with high-water marks and progress states:



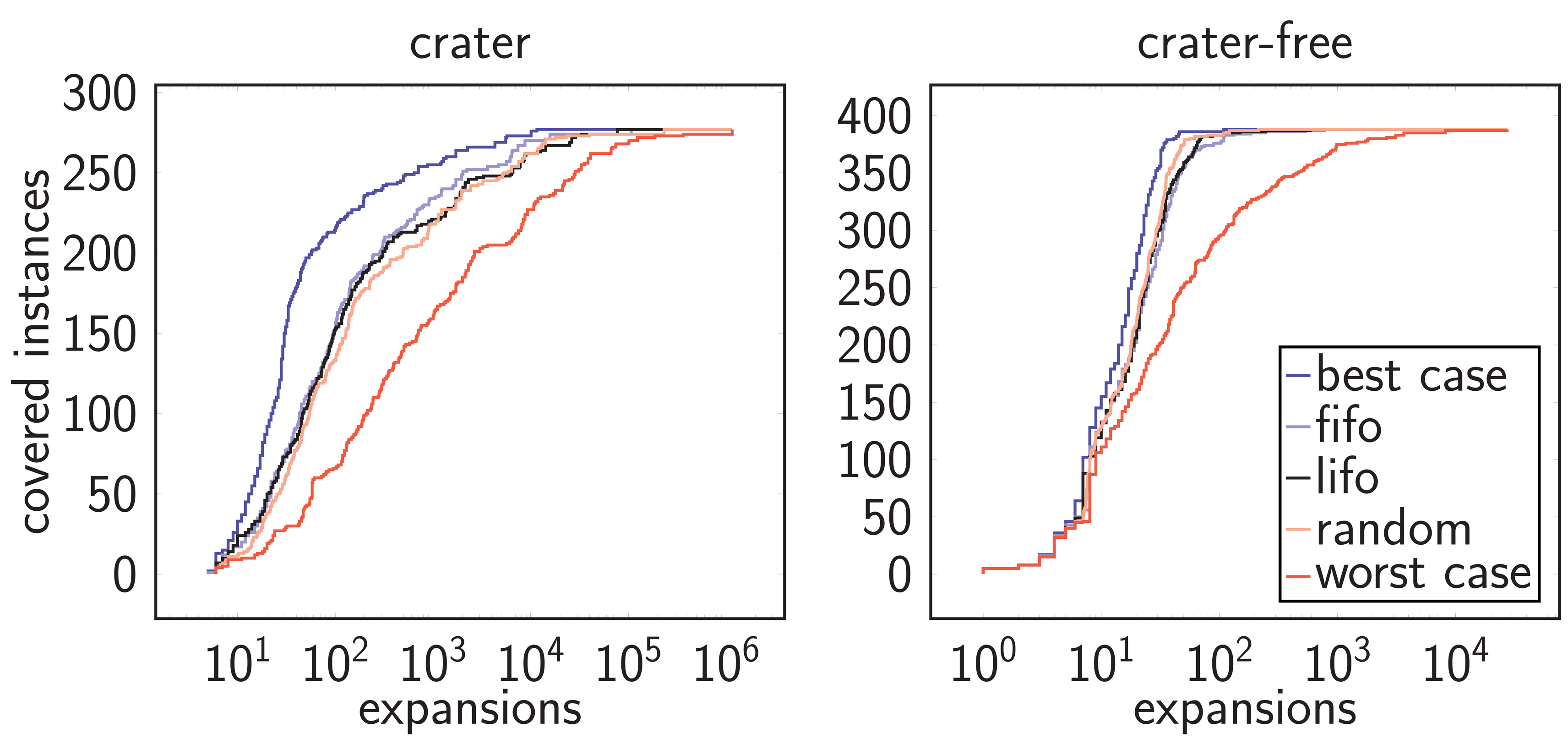
Bench transition system with benches, surface states and crater states:



Experimental Results

	best case				worst case			
	ov.-free	undir.	other	total	ov.-free	undir.	other	total
instances	406	31	327	764	471	10	283	764
covered	406	31	242	679	466	10	263	739

Comparison of standard tie-breaking strategies:



- Instances from classical planning tasks and planning heuristic

Conclusions

- Room for improvement over standard tie-breaking strategies, especially for weak heuristics
- Computing numbers for best case and worst case is often feasible
- Allows comparing the quality of heuristics for GBFS