Understandig the Search Behaviour of Greedy Best-First Search

Manuel Heusner Thomas Keller Malte Helmert

University of Basel

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Introduction

Open Questions

- Which states is GBFS guaranteed to expand?
- Which states is GBFS guaranteed not to expand?
- Which states may GBFS potentially expand?

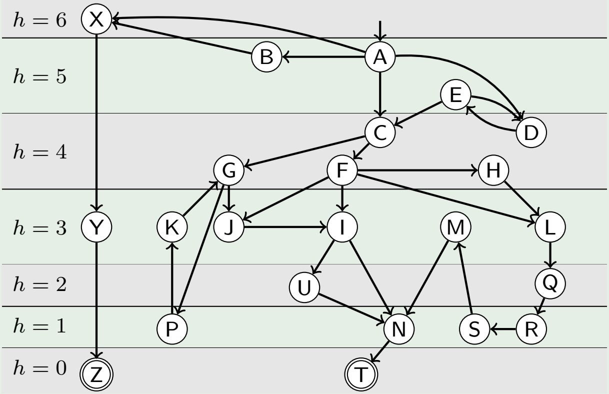
Note: Partly answered for A* (based on f-value) and for GBFS (based on high-water mark).

State Space Topology

- state space: generative model with initial state, goal states and successor function
- heuristic: assigns non-negative values to states
- state space topology: state space + heuristic

State Space Topology

Example

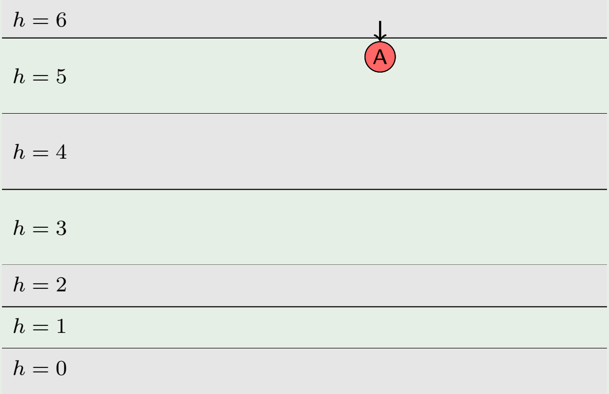


Greedy Best-First Search

- expansion : generates successors of a state
- greedy best-first search: iteratively expands states with lowest heuristic value
- tie-breaking: selects a state among states with equal heuristic values

Greedy Best-First Search

Example



Greedy Best-First Search

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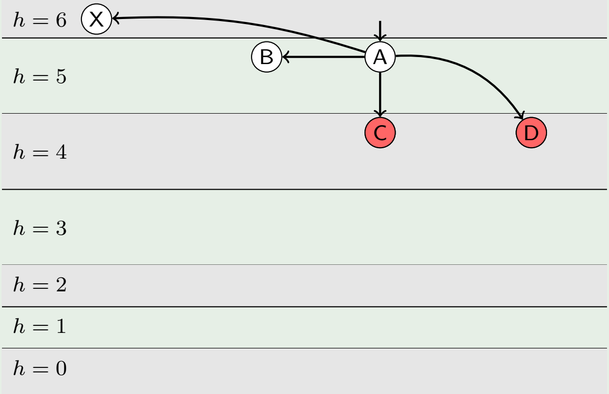

Greedy Best-First Search

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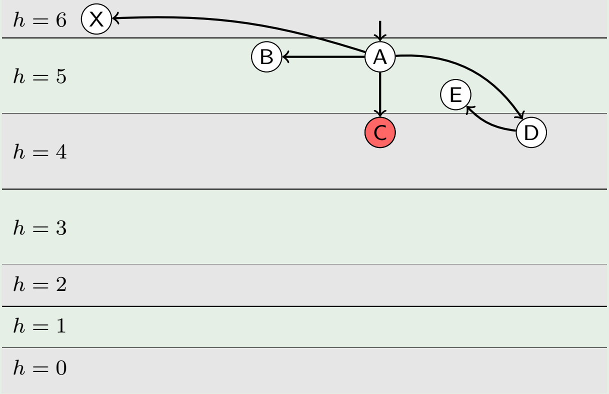

Greedy Best-First Search

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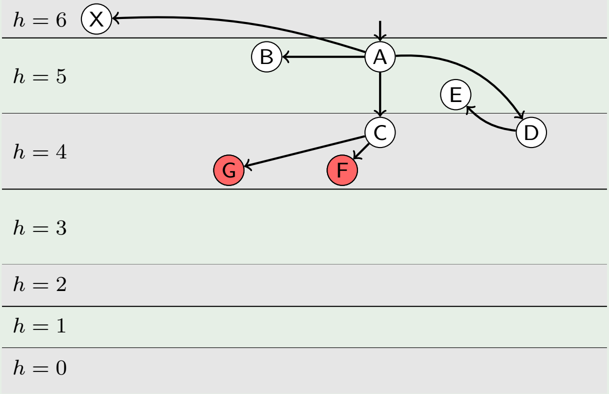

Greedy Best-First Search

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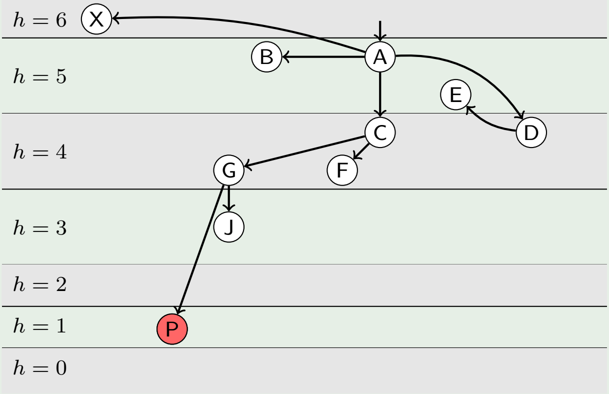
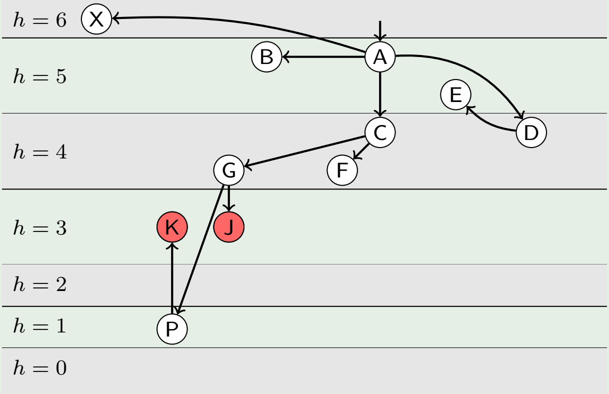


Greedy Best-First Search

Example



Greedy Best-First Search

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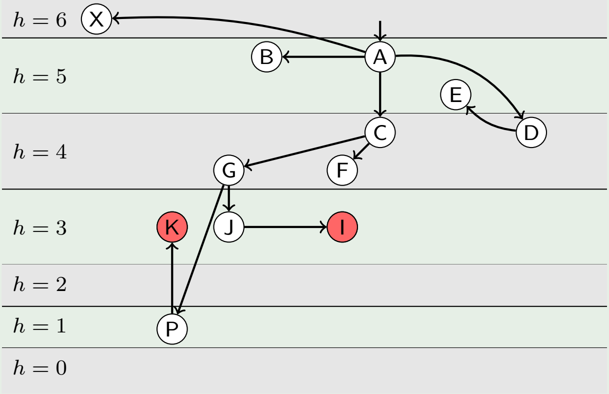
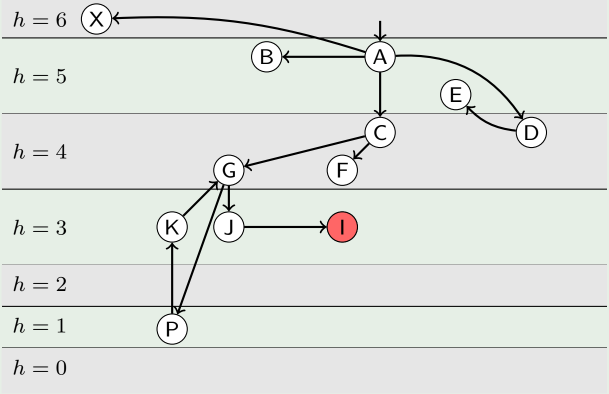


Greedy Best-First Search

Example



Greedy Best-First Search

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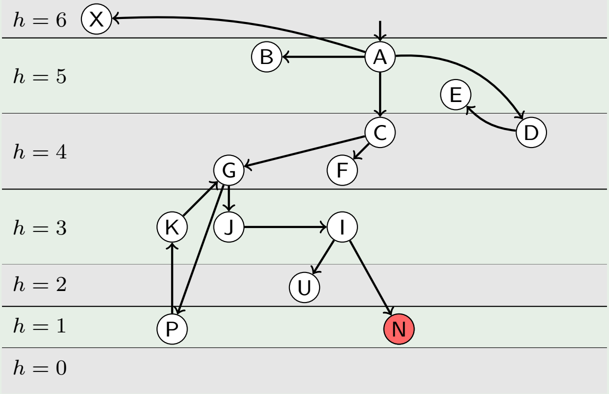

Greedy Best-First Search

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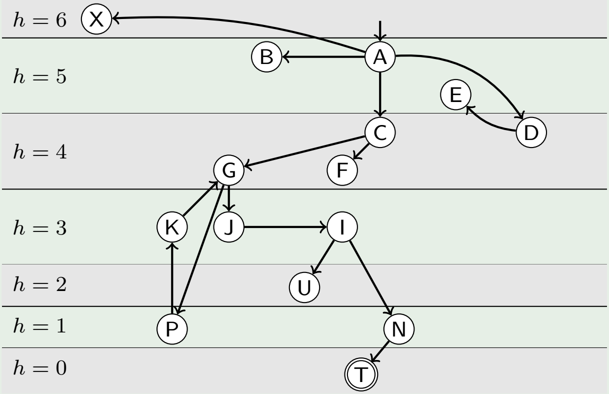


High-Water Marks

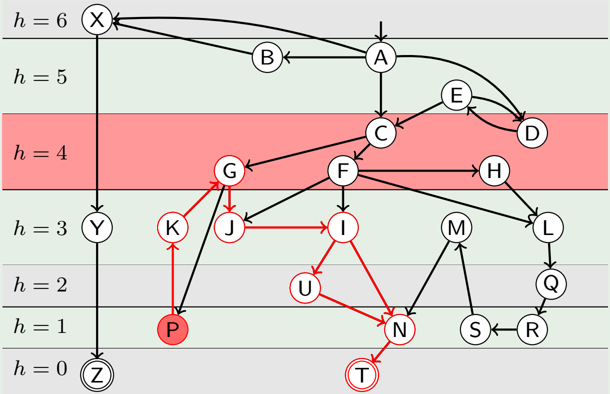
High-Water Marks

Definition (high-water mark)

The high-water mark is the largest heuristic value of a state that GBFS starting from a state (or a set of states) must expand before reaching a goal state.

High-Water Mark of State

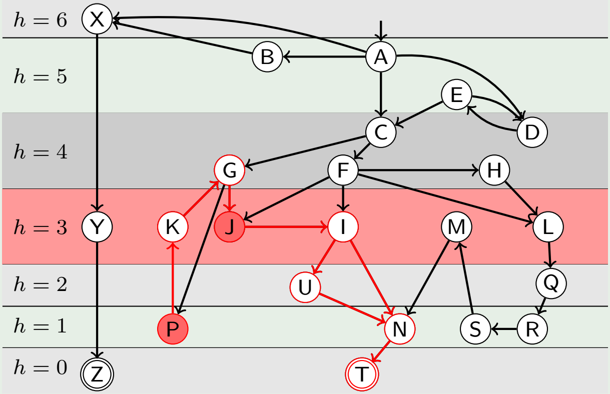
Example



high-water mark of state P: 4

High-Water Mark of State Set

Example



high-water mark of state set $\{J, P\}$: 3

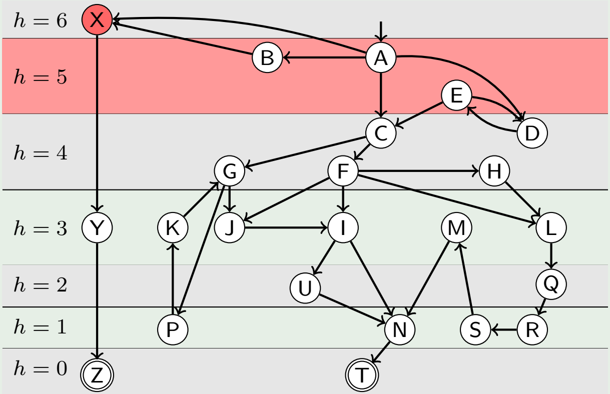
Earlier Result

Theorem (Wilt & Ruml, SoCS 2014)

GBFS is guaranteed to not expand a state whose heuristic value is larger than high-water mark of initial state.

Earlier Result

Example



never expanded states: $\{X\}$

Benches

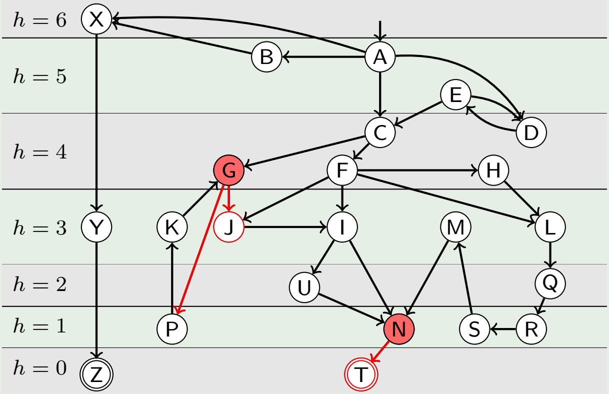
Bench Exit States

Definition (bench exit state)

Bench exit state is a state which has a successor that has lower high-water mark or that is a goal state.

Bench Exit States

Example



Bench Exit States

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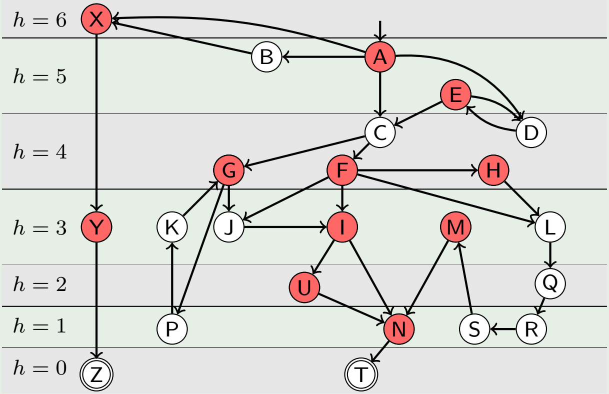


Bench Exit Property

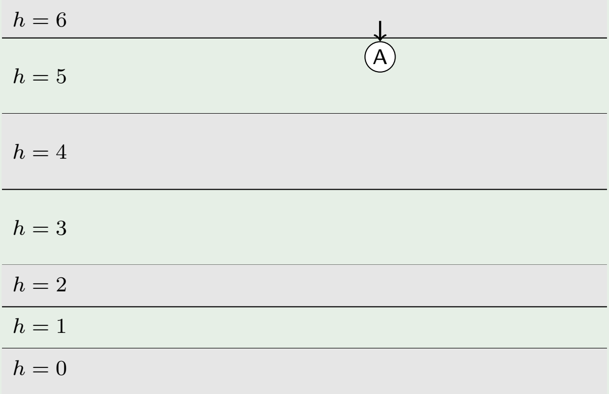
Theorem (bench exit property)

Whenever GBFS expands a bench exit state, all previously generated states will never be expanded for the rest of the algorithm run.

Note: GBFS makes progress when bench exit state is expanded.

Bench Exit Property

Example



Bench Exit Property

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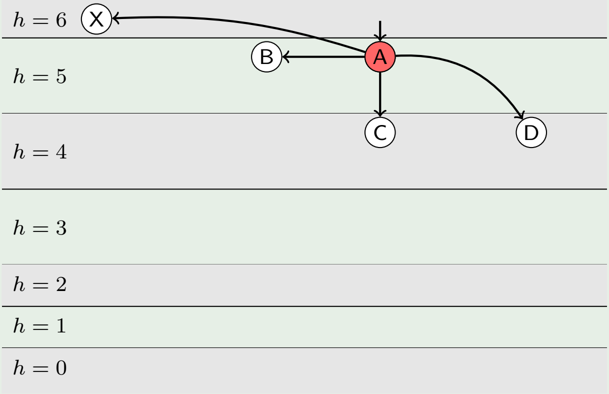

Bench Exit Property

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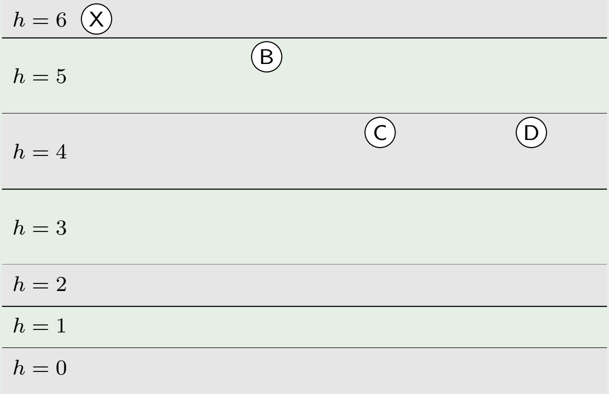

Bench Exit Property

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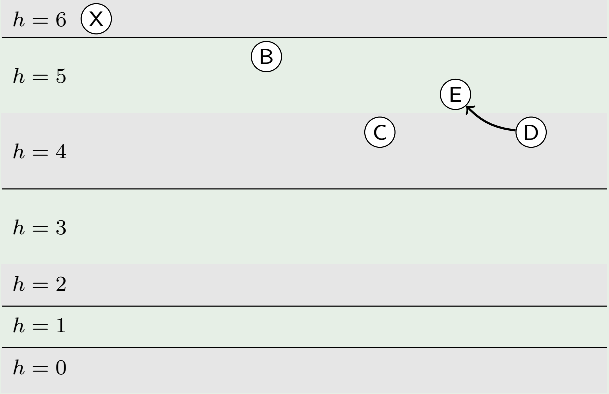

Bench Exit Property

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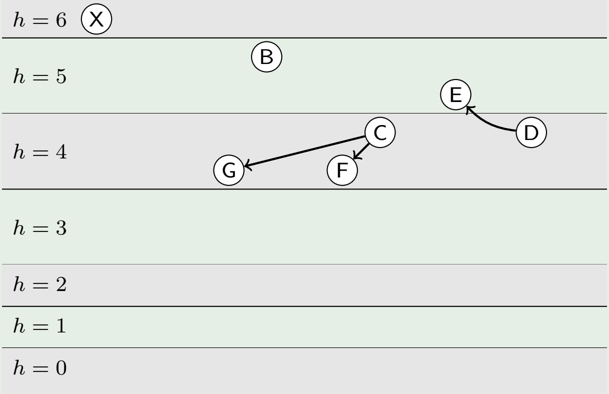

Bench Exit Property

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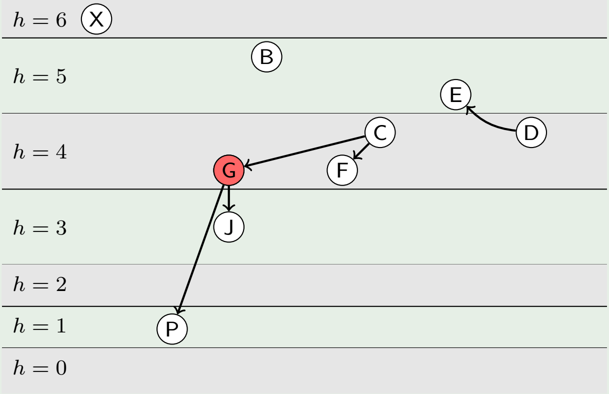

Bench Exit Property

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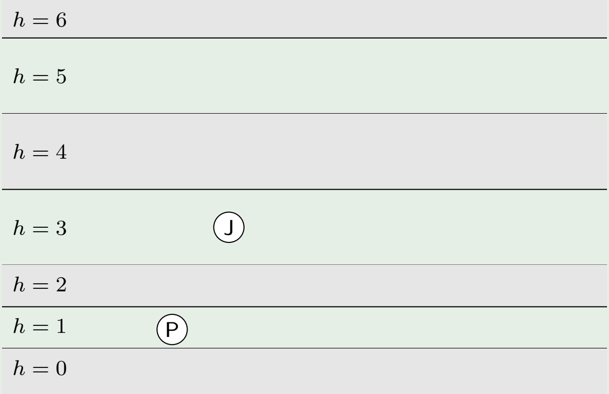

Bench Exit Property

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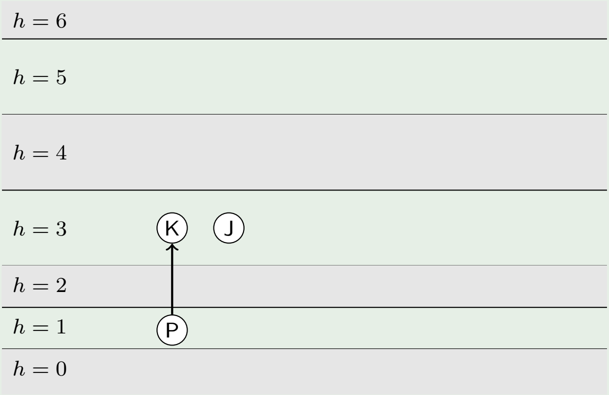

Bench Exit Property

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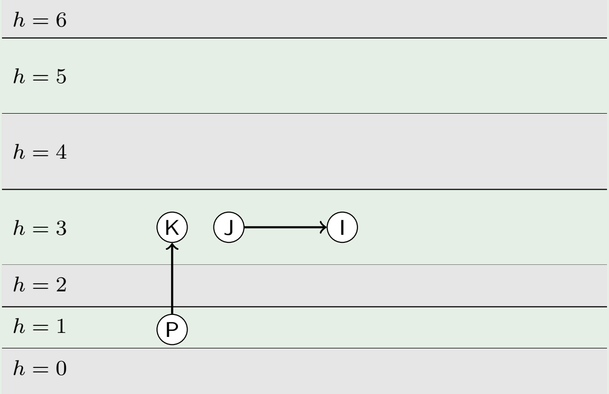

Bench Exit Property

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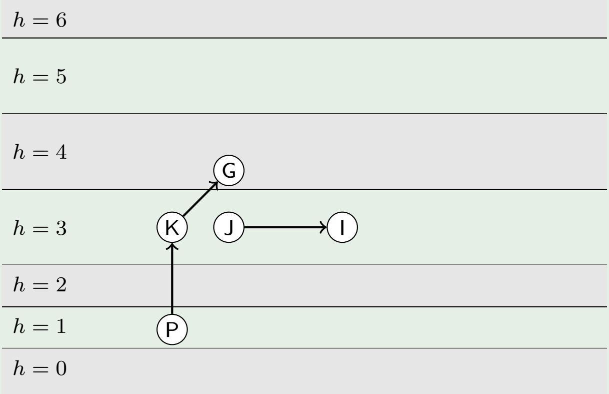

Bench Exit Property

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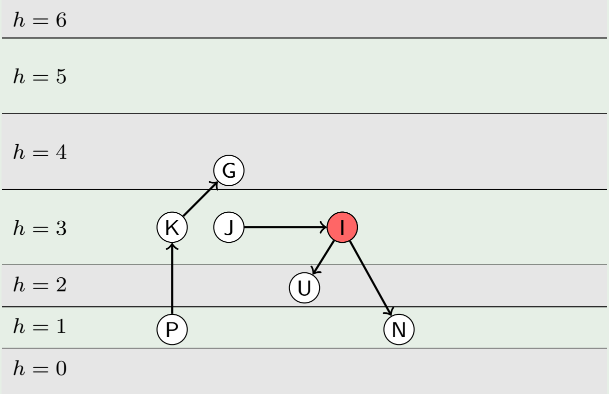

Bench Exit Property

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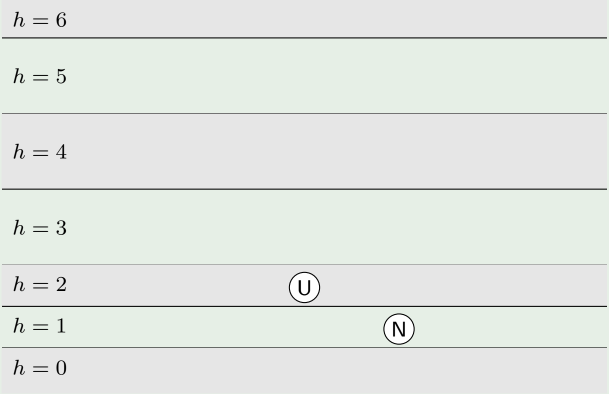

Bench Exit Property

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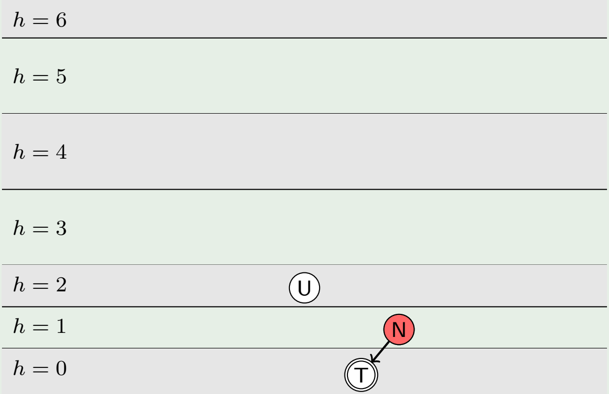


Bench Exit Property

Example

Benches

Definition (bench)

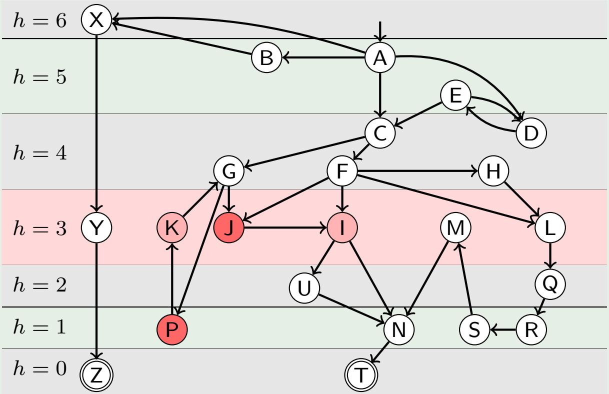
A bench contains all states that GBFS starting with a given set of states can expand until expansion of a bench exit state.

It is empty if the given set of states contains a goal state.

It is associated with high-water mark of the given set of states.

Benches

Example



states on bench defined by $\{J, P\}$: $\{I, J, P, K\}$

Bench Transition Systems

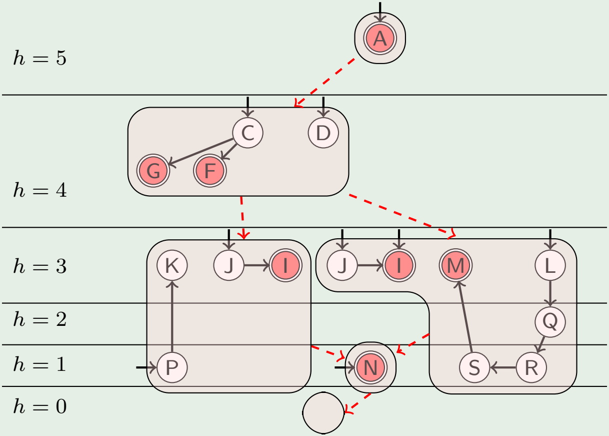
Definition (bench transition system)

A bench transition system contains all benches which are reachable from the bench that starts with the initial state.

A successor bench is defined by the successor states of a bench exit state.

Bench Transition Systems

Example



Results

Theorem

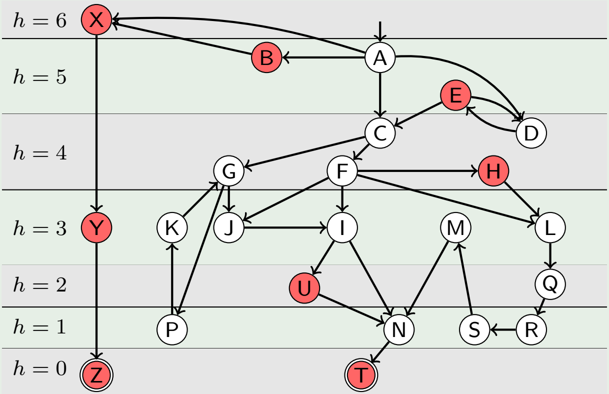
GBFS potentially expands a state if it is on at least one bench from bench transition system.

Theorem

GBFS is guaranteed to not expand a state that is not on a bench of the bench transition system.

Results

Example



never expanded states: $\{B, E, H, T, U, X, Y, Z\}$

Craters

Surfaces

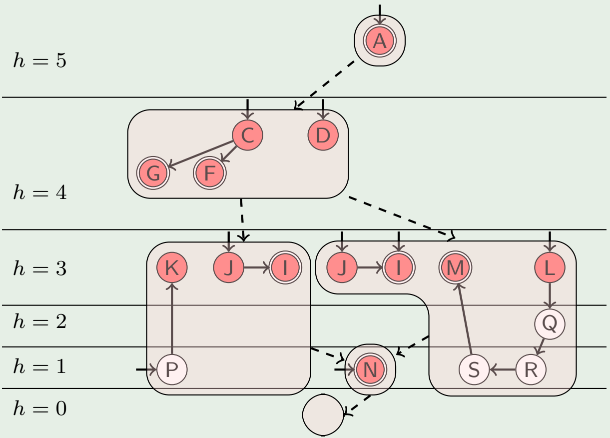
Definition (surface)

A state is on the surface of a bench if its heuristic value is the high-water mark of the bench.

Note: Is often called heuristic plateau or uninformed heuristic region.

Surfaces

Example



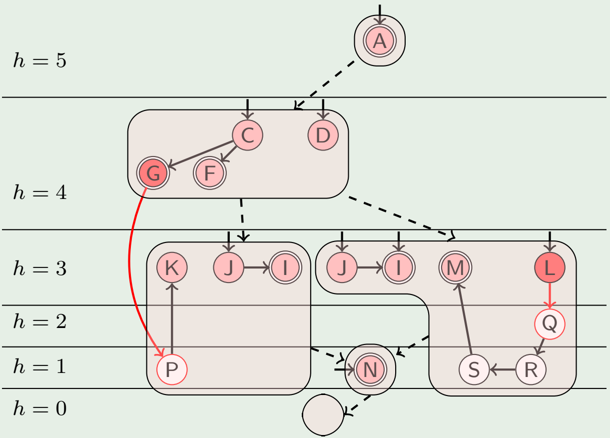
Crater Entry States

Definition (crater entry state)

A crater entry state is a state that is on the surface of a bench and that has a successor which is on a bench but not on a surface.

Crater Entry States

Example



Craters

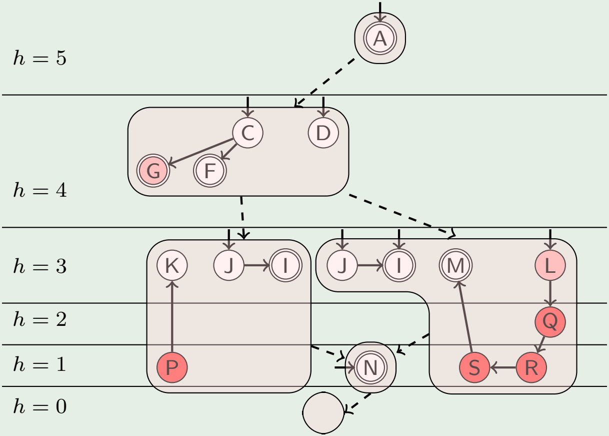
Definition (crater)

A crater contains all states that GBFS starting with a given crater entry state expands until expansion of a state from the surface.

Note: Is often called local minimum or uninformed heuristic region.

Craters

Example



Result

Theorem

Whenever GBFS expands a crater entry state s, then GBFS is guaranteed to expand all states in the crater defined by s.

Conclusion

Conclusion

- exact characterization of potentially expanded and never expanded states
- characterization of surely expanded states given some conditions
- better understanding of search behaviour and search progress