Introduction
General Operator Cost Partitioning
Relation to other Topics

From Non-Negative to General Operator Cost Partitioning

Florian Pommerening  Malte Helmert  Gabriele Röger
Jendrik Seipp

University of Basel, Switzerland

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Introduction
State space search

- Common approach: $A^*$ with admissible heuristic
- One heuristic often not sufficient
- How to combine heuristics?
State space search

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  - Sum?
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- Common approach: $A^*$ with admissible heuristic
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- How to combine heuristics?
  - Sum? Not admissible
  - Maximum? Does not use all information
State space search

- Common approach: $A^*$ with admissible heuristic
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- **How to combine heuristics?**
  - Sum? Not admissible
  - Maximum? Does not use all information

**Breakthrough:** **Cost partitioning**

- Make arbitrary heuristics additive
- Part of many state-of-the-art heuristics
Main idea

- Create copies of the original problem
- Distribute operator cost function between copies
- Compute one heuristic per copy
- Sum resulting heuristic values
Operator Cost Partitioning [Katz and Domshlak 2010]

Find cost functions $c_1, \ldots, c_n$ with

- Non-negative costs: $c_i \geq 0$
- Costs are distributed: $\sum_i c_i \leq$ original cost

$\Rightarrow$ Admissible estimates using cost function $c_i$ are additive
Operator Cost Partitioning

Find cost functions $c_1, \ldots, c_n$ with

- Non-negative costs: $c_i \geq 0$
- Costs are distributed: $\sum_i c_i \leq$ original cost

⇒ Admissible estimates using cost function $c_i$ are additive

Why restrict costs to non-negative values?
General Operator Cost Partitioning
General Operator Cost Partitioning

Find cost functions $c_1, \ldots, c_n$ with

- Non-negative costs: $c_i \geq 0$
- Costs are distributed: $\sum_i c_i \leq$ original cost

$\implies$ Admissible estimates using cost function $c_i$ are additive
General Cost Partitioning Example

Example

Heuristic value:
General Cost Partitioning Example

Example

- Heuristic value:
General Cost Partitioning Example

Heuristic value: $0 + 1 = 1$
General Cost Partitioning Example

Heuristic value: $0 + 2 = 2$
General Cost Partitioning Example

Heuristic value: $-\infty + 3 = -\infty$
Heuristic Quality of General Cost Partitioning

Expansions for optimal cost partitioning of atomic projections
Relation to Other Topics in Heuristic Search Planning
Operator-counting heuristics
State equation heuristic
A new approach to heuristic construction (potential heuristics)
1) Operator-Counting Heuristics

Operator-counting heuristics [Pommerening et al. 2014]

- Minimize total plan cost
- Subject to necessary properties of any plan (constraints)

Different sets of constraints define different heuristics
1) Operator-Counting Heuristics: Theoretical Result

Theorem

Combining operator-counting heuristics in one LP is equivalent to computing their optimal general cost partitioning.
2) State Equation Heuristic

Special case: state equation heuristic [van den Briel et al. 2007, Bonet 2013]

- Categorization previously unclear
  - Landmarks?
  - Abstractions?
  - Delete relaxations?
  - Critical paths?
2) State Equation Heuristic

Special case: state equation heuristic [van den Briel et al. 2007, Bonet 2013]

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Theorem

State equation heuristic

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Optimal general cost partitioning of all atomic projection heuristics
2) State Equation Heuristic

Special case: state equation heuristic [van den Briel et al. 2007, Bonet 2013]

- Categorization previously unclear
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Theorem

State equation heuristic

= Optimal general cost partitioning of all atomic projection heuristics
3) Potential Heuristics

**Potentials**

- Numerical value associated with each fact
- Heuristic value is **sum of potentials** for facts in state

Image credit: David Lapetina
3) Potential Heuristics

**Potentials**
- Numerical value associated with each fact
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Linear constraints over potentials
- **Express consistency and admissibility**
- **Necessary and sufficient conditions**
3) Potential Heuristics

Potentials

- Numerical value associated with each fact
- Heuristic value is sum of potentials for facts in state

Linear constraints over potentials

- Express consistency and admissibility
- Necessary and sufficient conditions

Optimization criterion

- Can optimize any function over potentials
- Here: maximize heuristic value of a state
3) Potential Heuristics: Theoretical Result

Theorem

Potential heuristic optimized in each state

= State equation heuristic

Optimizing potentials less frequently

- Trade off accuracy for evaluation speed
- Here: optimize once for heuristic value of initial state
3) Potential Heuristics: Practice

![Graph showing the number of solved tasks over time for different heuristics. The graph includes lines for max(Potential heuristics), Potential heuristic 1, Potential heuristic 2, and State equation heuristic. The x-axis represents time in seconds (10^-1 to 10^3), and the y-axis represents the number of solved tasks (200 to 700).]
Take Home Messages

Heuristic combination

Operator counting

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Optimal general cost partitioning

Equivalent heuristics

State equation heuristic

=

Optimal general cost partitioning of atomic projections

=

Potential heuristic (optimized in each state)

Interesting new heuristic family: potential heuristics
Potential Heuristics (Details)

Potential heuristic

Maximize $f(Potentials)$ subject to

$$\sum \text{Potential}_{\text{goal}}[V] \leq 0$$

$$\sum \text{Potential}_{\text{pre}(o)[V]} - \text{Potential}_{\text{eff}(o)[V]} \leq \text{cost}(o) \text{ for each } o \in O$$

Heuristic properties

- **Admissibility**: $h(s) \leq h^*(s)$ for all states $s$
- **Consistency**: $h(s) \leq h(s') + c(o)$ for all transitions $s \xrightarrow{o} s'$
- **Goal awareness**: $h(s) \leq 0$ for all goal states $s$
- **Goal awareness + consistency** $\iff$ **admissibility + consistency**