Higher-Dimensional Potential Heuristics for Optimal Classical Planning

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Higher-Dimensional Potential Heuristics for Optimal Classical Planning

- Find cheapest action sequence to achieve a goal.
- States are variable assignments.
- Operators change variable values.

Higher-Dimensional Potential Heuristics for Optimal Classical Planning

$$h(s) = \sum_{f \in \mathcal{F}} w(f)[s \models f]$$

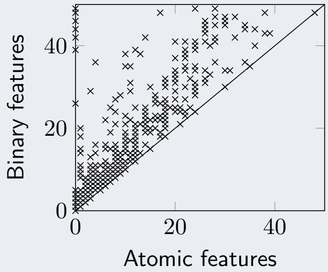
- Weighted sum of state features
- Two choices
  - Which features to use?
  - How to find good weights?

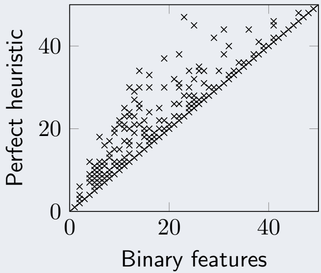
Higher-Dimensional Potential Heuristics for Optimal Classical Planning

- Features are conjunctions of facts
- Size of a feature: number of conjuncts
  - "Atomic" features (size 1)
    $w(\text{at-}A) = 10$, $w(\text{at-}B) = 5$
  - "Binary" features (size 2)
    $w(\text{at-}B \wedge \text{door-locked}) = 10$
  - ...

Why do we care about higher-dimensional features?

Initial heuristic values





- Atomic features are often not sufficient for high-quality heuristics

Goals

- Find good weights automatically
- Ideally:
  - Declare properties of heuristics (admissible, consistent)
  - Constraints characterize heuristics with these properties
  - Select best possible heuristic from the space of solutions

Our Contributions

Describing admissible and consistent potential heuristics

Features	Characterization
All atomic features	compact [previous work]
All binary features	compact [new]
All ternary features	intractable [new]
Subset of all features	fixed parameter tractable [new]

Also in the paper

- Potential functions $\simeq$ Transition cost partitioning

Compact Characterizations

Compact Characterization

Characterizing admissible and consistent heuristics

Goal awareness

$$h(s^*) \leq 0$$

- Easy: $h(s^*)$ is a sum of weights

Consistency

$$h(s) - h(s') \leq \mathit{cost}(o) \qquad \forall s \xrightarrow{o} s'$$

- Hard: exponential number of constraints

Consistency

- Consider a single operator
- Three types of features
  - irrelevant: no variables in common with o
  - context-independent: all variables in common with o
  - context-dependent: some in common with o, some not

Heuristic difference caused by operator o

$$h(s) - h(s') = \Delta_o^{\mathsf{irr}}(s) + \Delta_o^{\mathsf{ind}}(s) + \Delta_o^{\mathsf{dep}}(s)$$

Heuristic Difference when Applying Operator o

Consistency for an operator o

$$\Delta_o^{\text{irr}}(s) + \Delta_o^{\text{ind}}(s) + \Delta_o^{\text{dep}}(s) \leq \mathit{cost}(o) \qquad \forall s \xrightarrow{o} s'$$

Irrelevant features

- No variables in common with o
- No change in truth value when applying o
- Does not cause change in heuristic

Heuristic Difference when Applying Operator o

Consistency for an operator o

$$0 + \Delta_o^{\text{ind}}(s) + \Delta_o^{\text{dep}}(s) \leq \mathit{cost}(o) \qquad \forall s \xrightarrow{o} s'$$

Irrelevant features

- No variables in common with o
- No change in truth value when applying o
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Heuristic Difference when Applying Operator o

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Context-independent features

- All variables in common with o
- Change in truth value fully determined by o
- Heuristic change easy to specify and does not depend on state

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- Change in truth value fully determined by o
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Heuristic Difference when Applying Operator o

Consistency for an operator o

$$\Delta_o^{\text{ind}} + \Delta_o^{\text{dep}}(s) \leq cost(o) \qquad \forall s \xrightarrow{o} s'$$

Context-dependent features

- At least one variable in common with o
- At least one variable not mentioned by o
- Heuristic change depends on state

Context-Dependent Features

Context-Dependent Features

- Atomic features: no context-dependent features
- Binary features: context limited to one variable
  - “Worst value” exists for each variable
  - Worst case: all variables have worst value
  - Constraint for worst state implies all other constraints

Theorem

Admissible and consistent potential heuristics over binary features can be characterized by a compact set of linear constraints.

Larger Features

Intractability

In general

- Change in potential when applying o depends on more than one variable
- Influence of V on o depends on larger context

Theorem

Testing whether a given potential function is consistent is coNP-complete.
This already holds with only ternary features.

Proof:

- Reduction from non-3-colorability

Fixed Parameter Tractbility

Approach for binary features can be generalized

- Factor out influence of one variable at a time
- Generalization of Bucket Elimination algorithm from numerical cost functions to linear expressions

Theorem

Computing a set of linear constraints that characterize the admissible and consistent potential heuristics for a set of features is fixed-parameter tractable.
Parameter: tree-width of feature connectivity.

Take Home Messages

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Characterization of admissible and consistent potential functions

- Compact for binary features
- coNP-complete for ternary or larger features ...
- ... but fixed parameter tractable
  Parameter: tree-width of feature connectivity