

# Generalized Label Reduction for Merge-and-Shrink Heuristics

Silvan Sievers, Martin Wehrle and Malte Helmert

University of Basel, Switzerland

July 29, 2014



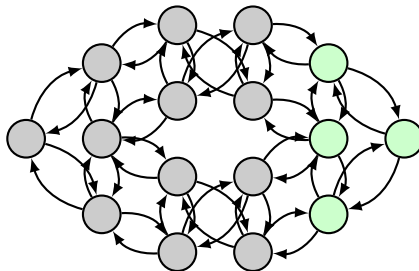
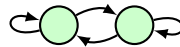
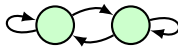
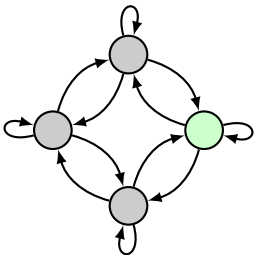
# Outline

- 1 Background
- 2 Generalized Label Reduction
- 3 Experiments

# Merge-and-Shrink Heuristics

- Distance heuristics for state space search (Dräger et al. (2006), Helmert et al. (2007), Nissim et al. (2011), Helmert et al. (2014))
- Idea:
  - Represent state space as set of small finite **automata**
  - State space corresponds to **product** of automata
  - **Transform** automata to obtain distance heuristic for state space
- Applicable for classical planning and **many other** state space search problems

# Example

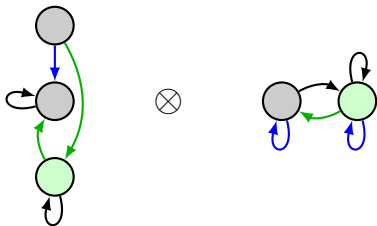


# Merge-and-Shrink Transformations (1)

- **Merge:** replace two automata by their product automaton

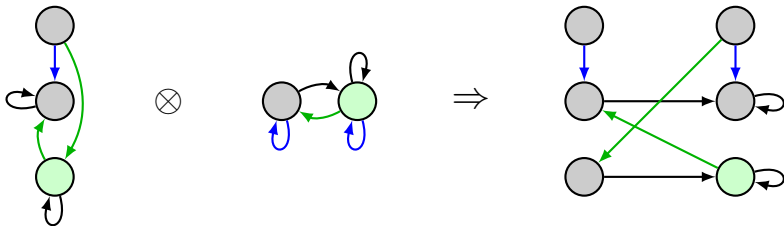
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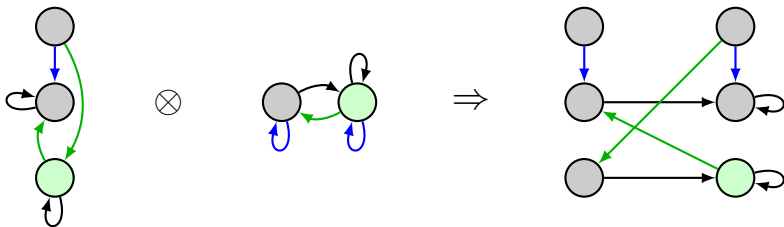
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- **Exact** transformation: preserves distances in represented state space

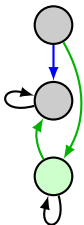


## Merge-and-Shrink Transformations (2)

- **Shrink:** abstract one automaton

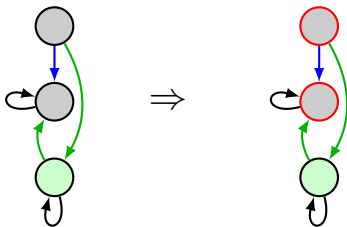
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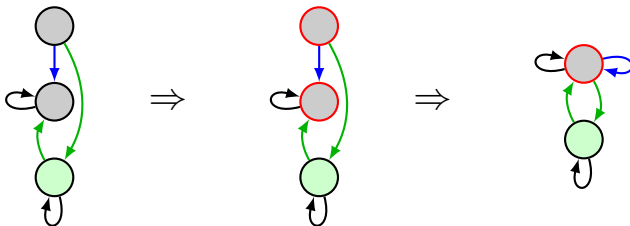
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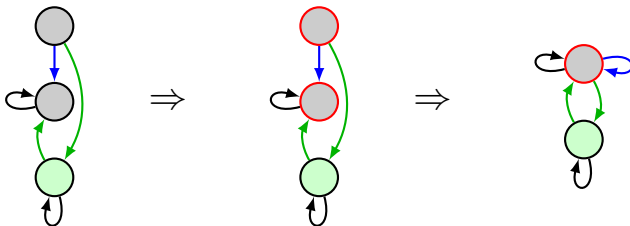
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- **Shrink:** abstract one automaton



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- **Shrink**: abstract one automaton



- **Safe** transformation: does not increase distances in represented state space  
(Exact with bisimulation, Nissim et al. (2011))

# Previous Label Reduction and its Flaws

## Proof Sketch for Theorem 5.11 of Helmert et al. (2014)

We prove by induction over the construction of  $T^\alpha$  that, for any intermediate merge-and-shrink abstraction  $\beta$  over  $V'$ :  $\Theta_\beta^\tau = \Theta^\beta$  if  $v^* \notin V'$ , and  $\Theta_\beta^\tau = \Theta^\beta|_{\tau V'}$  if  $v^* \in V'$ . The single tricky case in the induction is the case where  $\beta = \alpha_1 \otimes \alpha_2$  and (WLOG)  $v^* \in V_1$ . Using the induction hypothesis, we then need to prove that

$(\Theta^{\alpha_1}|_{\tau V_1} \otimes \Theta^{\alpha_2}|_{\tau V_1})|_{\tau V_1 \cup V_2} = \Theta^{\alpha_1 \otimes \alpha_2}|_{\tau V_1 \cup V_2}$ . Since  $\tau V_1$  is conservative for  $\Theta^{\pi V_1}$ , with  $V_2 \subseteq V_1$  and Proposition 5.4, it is conservative also for  $\Theta^{\alpha_2}$ . Hence, Lemma 5.6 reduces the left-hand side of our proof obligation to  $((\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau V_1})|_{\tau V_1 \cup V_2}$ , which with  $\tau V_1 \cup V_2 \circ \tau V_1 = \tau V_1 \cup V_2$  is equal to  $(\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau V_1 \cup V_2}$ . The claim then follows with Theorem 4.5.

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- Full potential restricted to **linear** merge strategies
- Based on **syntax** of underlying planning operators



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# Contribution

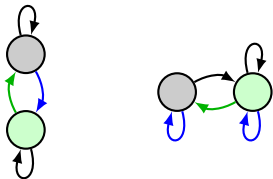
- Clear, easy and complete definition of label reduction
- Theoretic investigation: properties of label reduction (safeness and exactness)
- Empirical investigation for classical planning

# Generalized Label Reduction

- **Replace** all labels of a chosen set by one chosen **new label** in all automata

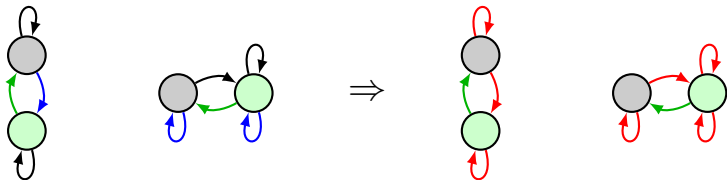
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# Theorem: Safeness

## Theorem

*Label reduction is **safe**, i. e. leaves the heuristic admissible.*

# Combinable Labels

## Definitions

- Labels are **locally equivalent** in automaton  $\Theta$  if they label the same set of transitions in  $\Theta$ .
- Labels are  **$\Theta$ -combinable** if they are locally equivalent in all automata but  $\Theta$ .
- Label  $l_1$  **globally subsumes** label  $l_2$  if the set of transitions labeled by  $l_2$  is a subset of the transitions labeled by  $l_1$  in all automata.

# Theorem: Exactness

## Theorem

A label reduction which maps labels  $l_1$  and  $l_2$  onto a new label  $l$  is *exact*, i. e. leaves the heuristic perfect, *if and only if*

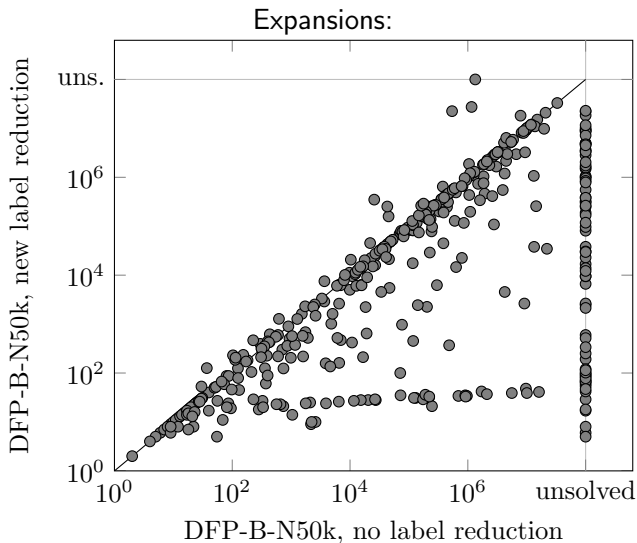
- 1  $l_1$  globally subsumes  $l_2$ , or
- 2  $l_2$  globally subsumes  $l_1$ , or
- 3  $l_1$  and  $l_2$  are  $\Theta$ -combinable for some automaton  $\Theta$  of the set of automata.



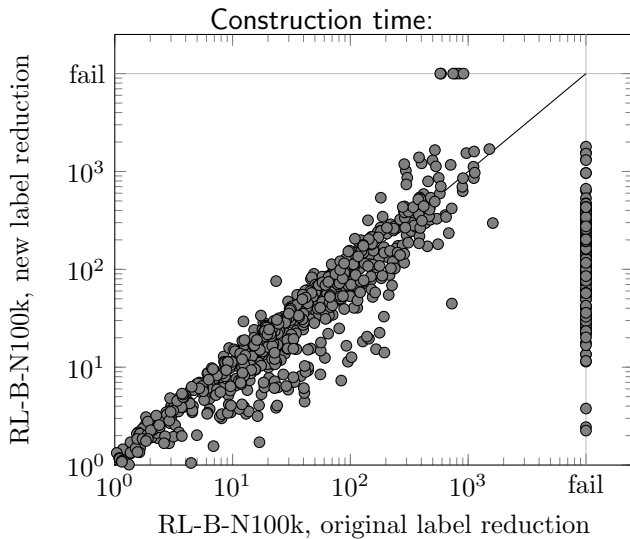
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## Results: Usefulness of Label Reduction



## Results: Old vs. New Label Reduction Method



# Conclusion

- **Generalized** label reduction for merge-and-shrink heuristics:
  - **Cleaner** and **easier** definition
  - **Safe** and unrestricted transformation
  - **Exact** transformation if based on  $\Theta$ -combinability
- Empirical **performance gain** for merge-and-shrink heuristics in classical planning
- Opened possibilities to develop even better merge-and-shrink heuristics

# The End

Thank you!

## Results: Coverage

Coverage:

merge/shrink strategy	Label Reduction		
	none	old	new
RL-B-N50k	577	618	634
RL-B-N100k	560	599	639
RL-B-N200k	544	590	630
DFP-B-N50k	565	—	644
DFP-B-N100k	551	—	632
DFP-B-N200k	522	—	625