

# An Empirical Case Study on Symmetry Handling in Cost-Optimal Planning as Heuristic Search

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# Motivation

- Successful usage of symmetries:
  - Planning: **duplicate pruning** in  $A^*$ , improved merge-and-shrink heuristics
  - Heuristic search: **symmetrical/dual lookups**

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  - Heuristic search: **symmetrical/dual lookups**
- Contribution of this work:
  - **Quantitative analysis** of symmetries in planning benchmarks
  - **Empirical comparison** of different symmetry-based techniques (adapted to planning)

# Outline

## 1 Background

## 2 Experiments

- Symmetries in Planning Benchmarks
- Symmetrical Lookups for Planning
- Comparison of Symmetry-based Techniques

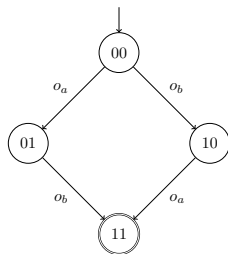
# Classical Planning

- SAS<sup>+</sup> planning task  $\Pi$ :
  - Finite-domain **state variables**
  - Initial state: complete variable assignment
  - Goal description: partial variable assignment
  - **Operators**: preconditions, effects, cost

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- State transition graph  $\mathcal{T}_\Pi$ :



# Structural Symmetries (Shleyfman et al. 2015)

- Structural symmetry of a planning task  $\Pi$ :
  - Maps **facts** (variable/value pairs) to facts and operators to operators
  - Induced symmetry  $\sigma$  on the state transition graph  $\mathcal{T}_\Pi = (V, E)$  is a **goal-stable automorphism**:
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    - $s$  goal state iff  $\sigma(s)$  goal state

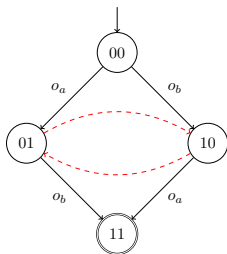
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- Example symmetry:

$$\sigma(o_a) = o_b$$

$$\sigma(o_b) = o_a$$



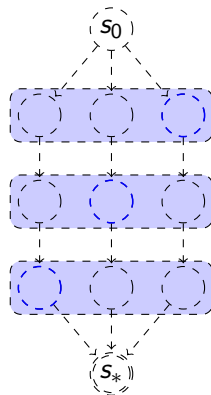


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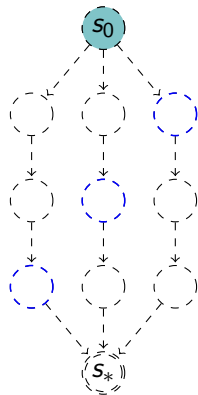
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Credits to A. Shleyfman

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  - When expanding state  $s$ , replace successors by **orbit representatives**, but save regular operators  
→ **symmetrical duplicate pruning**

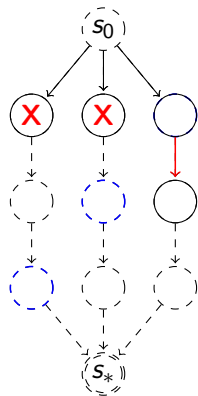


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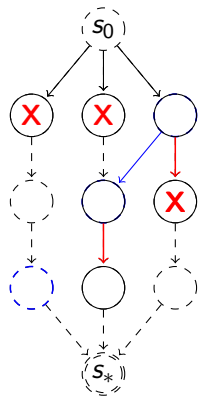


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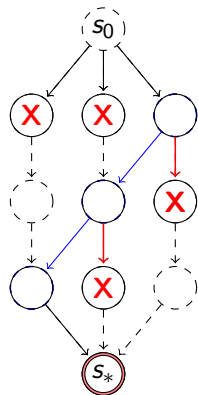
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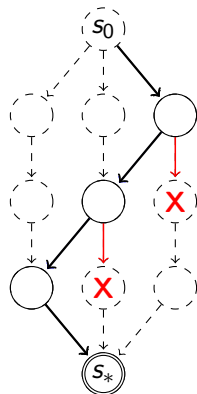
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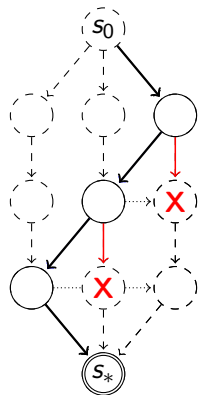
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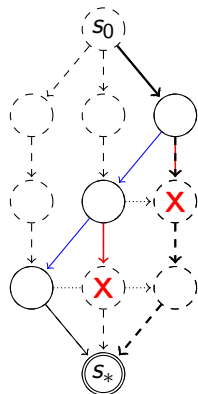
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# Symmetrical Lookups for Planning

- (For heuristic search: Felner et al. 2005, Zahavi et al. 2008)
- Before search: find (some) generators of the automorphism group
- During search, for a given state  $s$  and heuristic  $h$ :
  - Compute (a subset of) the orbit containing  $s$ :  
 $S := \{s, s^1, \dots, s^m\}$
  - Compute heuristic as  $\bar{h}(s) := \max\{h(s') \mid s' \in S\}$
- Properties:
  - $S$  can be chosen arbitrarily
  - $\bar{h}(s)$  is still admissible (if  $h$  is)

# Bidirectional Pathmax for Planning

- (For heuristic search: Felner et al. 2011)
- Symmetrical lookups usually render heuristics **inconsistent**
- Consistency:  $h(s) \leq cost(o) + h(s')$  for a transition from  $s$  to  $s'$  with operator  $o$
- **Bidirectional pathmax (BPMX) rule:**  
 $h(s') = \max(h(s'), h(s) - cost(o))$

# Merge-and-Shrink Heuristic (Helmert et al. 2014)

- Represent state space as set  $\mathcal{T}$  of small finite **transition systems**, with a **shared label set  $L$**
- State space corresponds to **product** of transition systems
- **Transform** transition systems to obtain distance heuristic for state space

## Factored Symmetries (Sievers et al. 2015)

- Work on a set  $\mathcal{T}$  of transition systems as encountered during the merge-and-shrink computation
- **Locally** map abstract states to abstract states within elements of  $\mathcal{T}$  and **globally** map transition labels to transition labels in  $L$
- Goal states must be preserved



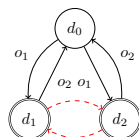
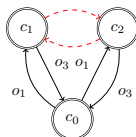
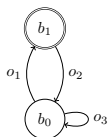
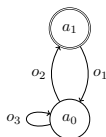
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- Example:

$$\sigma(o_1) = o_1$$

$$\sigma(o_2) = o_2$$

$$\sigma(o_3) = o_3$$



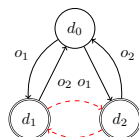
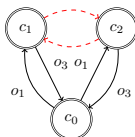
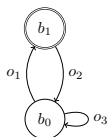
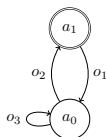
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- Usage: improve **merging strategies**

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# Quantitative Analysis

- Benchmark set: 44 domains with 1396 tasks
- Amount of symmetries:
  - Only 3 domains with no symmetries
  - 1103 tasks contain symmetries
  - In 38 domains, more than 50% of tasks contain symmetries
  - In most of the 38 domains, almost all tasks contain symmetries
- Influence of the representation and the symmetry tool?

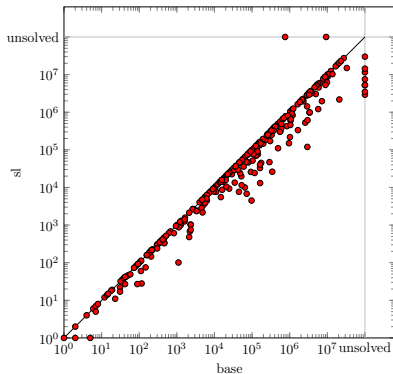
# Symmetrical Lookups

Merge-and-Shrink	base	1 state	5 states	10 states	orbit
Coverage	652	656	<b>658</b>	<b>658</b>	<b>658</b>
Expansions sum	607602428	501671723	493848579	<b>471769190</b>	493848579

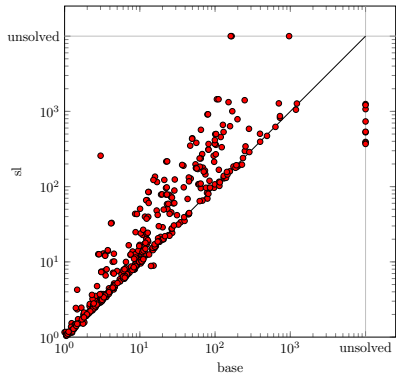
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Expansions:



Runtime:



# Bidirectional Pathmax

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Expansions sum	607602428	<b>471769190</b>	471769236

- **Marginal** reduction in expansions, no increase in coverage
- Explanation: pathmax corrections only in 2% of the tasks for which the merge-and-shrink heuristic was constructed

# Combinations of Techniques

<b>Merge-and-Shrink</b>	<b>base</b>	<b>oss</b>	<b>sl</b>	<b>fs</b>
<b>Coverage</b>	652	696	658	654
<b>Expansions sum</b>	5.16e+8	2.68e+8	4.01e+8	3.65e+8

- All techniques improve performance



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Merge-and-Shrink	oss-sl	oss-fs	sl-fs	all
Coverage	691	<b>698</b>	655	692
Expansions sum	2.54e+8	2.39e+8	3.44e+8	<b>2.32e+8</b>

- Including orbit space search **always helpful**
- Including symmetrical lookups **not very helpful** (for coverage)

# More Results ...

... on the poster!

# Conclusions

- Planning benchmarks contain **lots of symmetries**
- Symmetry-based techniques improve state-of-the-art planning techniques
- **Orbit space search** achieves best performance
- BMPX does not help as much as in heuristic search problems