A Compiler for Deterministic, Decomposable Negation Normal Form

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Abstract

We present a compiler for converting CNF formulas into deterministic, decomposable negation normal form (d-DNNF). This is a logical form that has been identified recently and shown to support a number of operations in polynomial time, including clausal entailment; model counting, minimization and enumeration; and probabilistic equivalence testing. d-DNNFs are also known to be a superset of, and more succinct than, OBDDs. The polytime logical operations supported by d-DNNFs are a subset of those supported by OBDDs, yet are sufficient for model-based diagnosis and planning applications. We present experimental results on compiling a variety of CNF formulas, some generated randomly and others corresponding to digital circuits. A number of the formulas we were able to compile efficiently could not be similarly handled by some state-of-the-art model counters, nor by some state-of-the-art OBDD compilers.

Introduction

A tractable logical form known as *Deterministic, Decomposable Negation Normal Form*, d-DNNF, has been proposed recently (Darwiche 2001c), which permits some generally intractable logical queries to be computed in time polynomial in the form size (Darwiche 2001c; Darwiche & Marquis 2001). These queries include clausal entailment; counting, minimizing, and enumerating models; and testing equivalence probabilistically (Darwiche & Huang 2002). Most notably, d-DNNF has been shown to be more succinct than OBDDs (Bryant 1986), which are now quite popular in supporting various AI applications, including diagnosis and planning. Moreover, although OBDDs are more tractable than d-DNNFs (support more polytime queries), the extra tractability does not appear to be relevant to some of these applications.

An algorithm has been presented in (Darwiche 2001a; 2001c) for compiling Conjunctive Normal Form (CNF) into d-DNNF. The algorithm is structure-based in two senses. First, its complexity is dictated by the connectivity of given CNF formula, with the complexity increasing exponentially with increased connectivity. Second, it is insensitive to non-structural properties of the given CNF: two formulas with the same connectivity are equally difficult to compile by the given algorithm. However, most CNF formulas of interest—including random formulas and those that arise in diagnosis, formal verification and planning domains—tend to have very high connectivity and are therefore outside the scope of this structure-based algorithm. Morever, some of these formulas can be efficiently compiled into OBDDs using state-of-the-art compilers such as CUDD. Given that d-DNNF is more succinct than OBDDs (in fact, d-DNNF is a strict superset of OBDD), such formulas should be efficiently compilable into d-DNNF too.

We present in this paper a CNF to d-DNNF compiler which is structure-based, yet is sensitive to the non-structural properties of a CNF formulas. The compiler is based on the one presented in (Darwiche 2001a) but incorporates a combination of additional techniques, some are novel, and others are well known in the satisfiability and OBDD literatures. Using the presented compiler, we show that we can successfully compile a wide range of CNF formulas, most of which have very high connectivity and, hence, are inaccessible to purely structure-based methods. Moreover, most of these formulas could not be compiled into OBDDs using a state-of-the-art OBDD compiler. The significance of the presented compiler is two fold. First, it represents the first CNF to d-DNNF compiler that practically matches the expectations set by theoretical results on the comparative succinctness between d-DNNFs and OBDDs. Second, it allows us to answer queries about certain CNF formulas that could not be answered before, including certain probabilistic queries about digital circuits.

Tractable forms: d-DNNF and OBDD

A negation normal form (NNF) is a rooted directed acyclic graph in which each leaf node is labeled with a literal, $true$ or $false$, and each internal node is labeled with a conjunction $\wedge$ or disjunction $\vee$. Figure 1 depicts an example. For any node n in an NNF graph, $Vars(n)$ denotes all propositional variables that appear in the subgraph rooted at n, and $\Delta(n)$ denotes the formula represented by n and its descendants. A number of properties can be stated on NNF graphs:

- Decomposability holds when $Vars(n_i) \cap Vars(n_j) = \emptyset$ for any two children n_i and n_j of an and-node n. The NNF in Figure 1 is decomposable.
- Determinism holds when $\Delta(n_i) \wedge \Delta(n_j)$ is logically in-

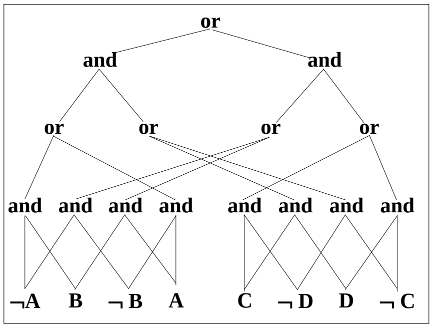
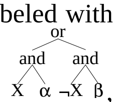
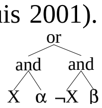
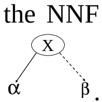


Figure 1: A negation normal form graph.

consistent for any two children n_i and n_j of an or-node n. The NNF in Figure 1 is deterministic.

- Decision holds when the root node of the NNF graph is a decision node. A *decision node* is a node labeled with $true$, $false$, or is an or-node having the form , where X is a variable, α and β are decision nodes. Here, X is called the *decision variable* of the node. The NNF in Figure 1 does not satisfy the decision property since its root is not a decision node.

- Ordering is defined only for NNFs that satisfy the decision property. Ordering holds when decision variables appear in the same order along any path from the root to any leaf.

Satisfiability and clausal entailment can be decided in linear time for decomposable negation normal form (DNNF) (Darwiche 2001a). Moreover, its models can be enumerated in output polynomial time, and any subset of its variables can be forgotten (existentially quantified) in linear time. Deterministic, decomposable negation normal form (d-DNNF) is even more tractable as we can count its models given any variable instantiation in polytime (Darwiche 2001c; Darwiche & Marquis 2001). Decision implies determinism. The subset of NNF that satisfies decomposability and decision (hence, determinism) corresponds to Free Binary Decision Diagrams (FBDDs) (Gergov & Meinel 1994). The subset of NNF that satisfies decomposability, decision (hence, determinism) and ordering corresponds to Ordered Binary Decision Diagrams (OBDDs) (Bryant 1986; Darwiche & Marquis 2001). In OBDD notation, however, the NNF fragment  is drawn more compactly as . Hence, each non-leaf OBDD node generates three NNF nodes and six NNF edges.

Immediate from the above definitions, we have the following strict subset inclusions OBDD $\subset$ FBDD $\subset$ d-DNNF $\subset$ DNNF. Moreover, we have OBDD $>$ FBDD $>$ d-DNNF $>$ DNNF, where $>$ stands for "less succinct than."[1] OBDDs are more tractable than DNNF, d-DNNF and FBDD. General entailment among OBDDs can be decided in polytime. Hence, the equivalence of two OBDDs can be decided in polytime. The equivalence of two DNNFs cannot be decided in polytime (unless P=NP). The equivalence question is still open for d-DNNF and FBDD, although both support polynomial probabilistic equivalence tests (Blum, Chandra, & Wegman 1980; Darwiche & Huang 2002). For a comprehensive analysis of these forms, the reader is referred to (Darwiche & Marquis 2001).

We close this section by noting that the polytime operations supported by DNNF are sufficient to implement model-based diagnosers whose complexity is linear in the size of compiled device, assuming the device is expressed as a DNNF (Darwiche 2001a). Moreover, for planning and formal verification applications, there is no need for a polytime test for general entailment as long as the goal (or property to be verified) can be expressed as a CNF (clausal entailment can be used here). Finally, polytime equivalence testing is not needed here as it is only used to check for fixed points: whether the models of some theory Δ^t (reachable states at time t) equal the models of some theory Δ^{t+1} (reachable states at time $t+1$), where $\Delta^t \models \Delta^{t+1}$ (the states reachable at t are included in those reachable at $t+1$). The two theories are equivalent in this case iff they have the same number of models. Hence, counting models is sufficient to detect fixed points in these applications.

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CNF2DDNNF(n, Ω)
 if n is a leaf node, return CLAUSE2DDNNF(Clauses(n) | Ω)
 ψ←CNF2KEY(Clauses(n) | Ω)
 if CACHE_n(ψ) ≠ NIL, return CACHE_n(ψ)
 Γ←CASE_ANALYSIS(n, Ω)
 CACHE_n(ψ)←Γ
 return Γ

CASE_ANALYSIS(n, Ω)
 Σ←Sep(n, Ω)
 if Σ = ∅, return CONJOIN(CNF2DDNNF(n_l, Ω),CNF2DDNNF(n_r, Ω))
 X← choose a variable in Σ
 WHILE_CASE(X,true,Π):
    if Π = ∅, α+←false
    else α+←CONJOIN(Π, CASE_ANALYSIS(n, Π ∪ Ω))
 WHILE_CASE(X,false,Π):
    if Π = ∅, α−←false
    else α−←CONJOIN(Π, CASE_ANALYSIS(n, Π ∪ Ω))
 return DISJOIN(α+,α−)
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Figure 2: Compiling a CNF into d-DNNF.

Compiling CNF into d-DNNF

Figure 2 depicts the pseudocode of an algorithm for compiling a CNF into a d-DNNF. The presented algorithm uses a

[1] That DNNF is strictly more succinct than d-DNNF assumes the non-collapse of the polynomial heirarchy (Darwiche & Marquis 2001).

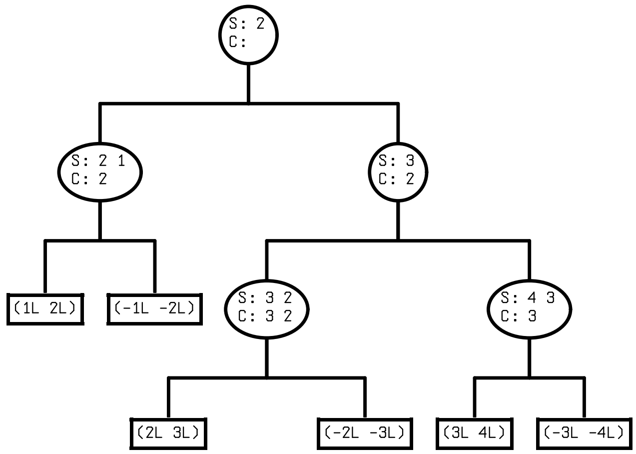

Figure 3: A decomposition tree for a CNF. Each leaf node is labeled with a clause (a set of literals). Each internal node is labeled with a separator (S) and a context (C).

data structure, known as a *decomposition tree (dtree)*, which is a full binary tree with its leaves corresponding to clauses in the given CNF (Darwiche 2001a). Figure 3 shows an example dtree, where each leaf node is labeled with a clause and each internal node is labeled with two sets of variables to be explained later. The algorithm works as follows. Each node n in the dtree corresponds to the set of clauses, $Clauses(n)$, appearing in the subtree rooted at n. Let n_l and n_r denote the left and right children of node n. If $Clauses(n_l)$ and $Clauses(n_r)$ do not share variables, then we can convert $Clauses(n_l)$ into a d-DNNF α_l and $Clauses(n_r)$ into a d-DNNF α_r and simply return $\alpha_l \wedge \alpha_r$ as the d-DNNF of $Clauses(n)$. In general, $Clauses(n_l)$ and $Clauses(n_r)$ do share variables, called a *separator* for dtree node node n . In that case, we choose one of these variables, call it X, and then perform a case analysis on it.

Case analysis. To perform case analysis on a variable X is to consider two cases, one under which X is set to true and another under which it is set to false. Under each case, X is eliminated from the given set of clauses. If α^+ is the result of converting $Clauses(n)$ into d-DNNF under $X = true$, and if α^- is the result of converting $Clauses(n)$ into d-DNNF under $X = false$, then $(X \wedge \alpha^+) \vee (\neg X \wedge \alpha^-)$ is a d-DNNF equivalent to $Clauses(n)$.[2] Case analysis is implemented using the macro WHILE_CASE(X,v,Π) on Lines 10 & 13, which replaces every occurrence of the variable X by v, performs unit resolution, and then collects all derived literals (including $X = v$) in Π. Note here that Π not only contains the literal $X = v$ as suggested above, but also all other literals derived by unit resolution (this leads to better results in general). If unit resolution derives a contradiction, Π is then the empty set.

[2] This is known as the Shannon expansion of $Clauses(n)$ in the literature on Boolean logic. It was initially proposed by Boole, however (Boole 1848).

Separators. We may have to perform case analysis on more than one variable before we can decompose $Clauses(n_l)$ and $Clauses(n_r)$; that is, before we eliminate every common variable between them. In general though, we do not need to perform case analysis on every variable common between $Clauses(n_l)$ and $Clauses(n_r)$. By setting a variable X to some value, some clauses under n_l or n_r may become subsumed, hence, eliminating some more variables that are common between them. This is why the separator for node n is defined with respect to a set of literals Ω on Line 7. That is, $Sep(n, \Omega)$ is defined as the variables common between $Clauses(n_l) \mid \Omega$ and $Clauses(n_r) \mid \Omega$, where $Clauses(.) \mid \Omega$ is the result of *conditioning* the clauses . on the literals Ω. That is, $Clauses(.) \mid \Omega$ is the set of clauses which results from eliminating the variables in Ω from . and replacing them by either true or false according to their signs in Ω.[3] Figure 3 depicts the separator for each node the given dtree, assuming $\Omega = \emptyset$.

The choice of which variable to set next from the separator $Sep(n, \Omega)$ on Line 9 has an effect on the overall time to compile into d-DNNF and also on the size of resulting d-DNNF. In our current implementation, we choose the variable that appears in the largest number of binary clauses. Finally, the base case in the recursive procedure of Figure 2 is when we reach a leaf node n in the dtree (Line 1), which means that $Clauses(n)$ contains a single clause. In this case, CLAUSE2DDNNF(.) is a constant time procedure which converts a clause into a d-DNNF.[4]

Unique nodes. Another technique we employ comes from the literature on OBDDs and is aimed at avoiding the construction of redundant NNF nodes. Two nodes are redundant if they share the same label (disjunction or conjunction) and have the same children. To avoid redundancy, we cache every constructed NNF node, indexed by its children and label. Before we construct a new NNF node, we first check the cache and construct the node only if no equivalent node is found in the cache. This technique is implicit in the implementation of CONJOIN and DISJOIN.[5]

Caching. Probably the most important technique we employ comes from the literature on dynamic programming. Specifically, each time we compile $Clauses(n) \mid \Omega$ into a d-DNNF α, we store (the root of) d-DNNF α in a cache associated with dtree node n; see Line 5. When the algorithm tries to compile $Clauses(n) \mid \Omega$ again, the cache associated with node node n is first checked (Lines 2&3). The *cache key* we use to store the d-DNNF α is a string generated from $Clauses(n) \mid \Omega$: each non-subsumed clause in $Clauses(n) \mid \Omega$ has two characters, one capturing its identity and the other capturing its literals. The generation of such a key is expensive, but the savings introduced by this

[3] This process is also known as *restriction* in the literature on Boolean logic.

[4] A clause $l_1, \ldots, l_m$ can be converted into a d-DNNF as follows: $\bigvee_{i=1}^{m} l_i \bigwedge_{j=1}^{i-1} \neg l_j$.

[5] CONJOIN and DISJOIN will construct nodes with multiple children when possible. For example, when conjoining two conjunctions, CONJOIN will generate one node labeled with $\wedge$ and have it point to the children of nodes being conjoined.

caching scheme are critical. This caching scheme is a major improvement on the one proposed in (Darwiche 2001a; 2001c). In the cited work, a *context* for node n, $Context(n)$, is defined as the set of variables that appear in the separator of some ancestor of n and also in the subtree rooted at n; see Figure 3. It is then suggested that d-DNNF α of $Clauses(n) \mid \Omega$ be cached under a key, which corresponds to the subset of literals Ω pertaining to the variables in $Context(n)$. That is, if $Clauses(n) = \{A \vee \neg B, C \vee D\}$, then $Clauses(n) \mid \{A\}$ could be cached under key A, and $Clauses(n) \mid \{\neg B\}$ could be cached under key $\neg B$, hence, generating two different subproblems. Using our caching approach, both $Clauses(n) \mid \{A\}$ and $Clauses(n) \mid \{\neg B\}$ will generate the same key, and will be treated as instances of the same subproblem, since both are equivalent to $\{C \vee D\}$.

Constructing dtrees. Another major factor that affects the behavior of our algorithm is the choice of a dtree. At first, one may think that we need to choose a dtree where the sizes of separators are minimized. As it turns out, however, this is only one important factor which needs to be balanced by minimizing the size of contexts as defined above. The smaller the separators, the fewer case analyses we have to consider. The smaller the contexts, the higher the cache hit rate. Unfortunately, these two objectives are conflicting: dtrees with small separator tend to have large contexts and the other way around. A better paramater to optimize is the size of clusters. The cluster of node n is the union of its separator and context. The size of the maximum cluster -1 is known as the *dtree width* (Darwiche 2001a). In our current implementation, we construct dtrees using the method described in (Darwiche & Hopkins 2001), which is based on recursive hypergraph decomposition. Specifically, the given CNF Δ is converted into a hypergraph G, where each clause in Δ is represented as a *hypernode* in G. Each variable X in CNF Δ is then represented as a *hyperedge* in G, which connects all hypernodes (clauses) of G in which X appears. Once the hypergraph G is constructed, we partition it into two pieces G_l and G_r, hence, partitioning the set of clauses in Δ into two corresponding sets Δ_l and Δ_r. This decomposition corresponds to the root of our dtree, and the process can be repeated recursively until the set of clauses in Δ are decomposed into singletons. Hypergraph decomposition algorithms try to attain two objectives: minimize the number of hyperedges that cross between G_l and G_r, and balance the sizes of G_l and G_r. These two objectives lead to generating dtrees with small widths as has been shown in (Darwiche & Hopkins 2001). The construction of a dtree according to the above method is quite fast and predictable, so we don't include the time for converting a CNF into a dtree in the experimental results to follow. We have to mention two facts though about the method described above. First, the hypergraph partitioning algorithm we use is randomized, hence, it is hard to generate the same dtree again for a given CNF. This also means that there is no guarantee that one would obtain the same d-DNNF for a given CNF, unless the same dtree is used across different runs. Second, the hypergraph partitioning algorithm requires a balance factor, which is used to enforce the balance constraint. We have found that a balance factor of 3/1 seems to generate good results in general. Therefore, if one does not have time to search across different balance factors, a balance factor of 3/1 is our recommended setting.

We close this section by noting that to compile a CNF Δ into a d-DNNF, we have to first construct a dtree with root n for Δ and then call CNF2DDNNF$(n, \emptyset)$.

Experimental results

We will now apply the presented CNF2DDNNF compiler to a number of CNFs. The experiment were run on a Windows platform, with a 1GHz processor. Our implementation is in LISP! We expect a C implementation to be an order of magnitude faster. The compiler is available through a web interface—please contact the author for details.

Random CNF formulas

Our first set of CNFs comes from SATLIB[6] and includes satisfiable, random 3CNF formulas in the crossover region, in addition to formulas corresponding to graph coloring problems; see Table 1.[7] Random 3CNF formulas (uf50–uf200) could be easily compiled with less than a minute on average for the largest ones (200 vars). Compiling such CNFs into OBDDs using the state-of-the-art CUDD[8] compiler was not feasible in general.[9] For example, we could not compile the first instance of uf100 within four hours. Moreover, the first instance in uf50 takes about 20 minutes to compile. More than 2 million nodes are constructed in the process, with more than 500 thousand nodes present in memory at some point (the final OBDD has only 82 nodes though). We have to point out here that we used CUDD in a straightforward manner. That is, we simply constructed an OBDD for each clause and then conjoined these clauses according to their order in the CNF. There are more sophisticated approaches for converting CNFs into OBDDs that have been reported recently (Aloul, Markov, & Sakallah 2001; December 2001). No experimental results are available at this stage, however, on compiling random CNFs into OBDDs using these approaches. We will report on these approaches with respect to other datasets later on though.

We also report on the compilation of graph coloring problems in Table 1 (flat100 and flat200). As is clear from the table, these CNFs can be easily compiled into small d-DNNFs that have a large number of models. Each one of these models is a graph coloring solution. Not only can we count these solutions, but we can also answer a variety of queries about these solutions in linear time. Examples: How many solutions set the color of node n to c? Is it true that when node n_1 is assigned color c_1, then node n_2 must be assigned color c_2? And so on? Although compiling a flat200 CNF takes 11 minutes on average, answering any of the previous queries can be done by simply traversing the compiled d-DNNF only once (Darwiche 2001c), which takes less than a

[6] *http://www.intellektik.informatik.tu-darmstadt.de/SATLIB/*

[7] Sets uf50 and uf100 contain 1000 instances each. We only use the first 100 instances.

[8] *http://vlsi.colorado.edu/ fabio/CUDD/*

[9] We used the sift-converge dynamic ordering heuristic in our experiments.

Name	Vars/Clause	d-DNNF nodes	d-DNNF edges	Model count	Time (sec)
uf50	50/218	111	258.4	362.2	1
uf100	100/430	1333.3	4765.3	1590706.1	2
uf150	150/645	3799.8	15018.5	68403010	8
uf200	200/860	4761.8	19273.3	1567696500	37
flat100	300/1117	1347.2	8565.2	8936035	4
flat200	600/2237	4794.9	46951.3	2.2202334e+13	636

Table 1: CNF benchmarks from SATLIB. Each set contains a 100 instances. We report the average over all instances.

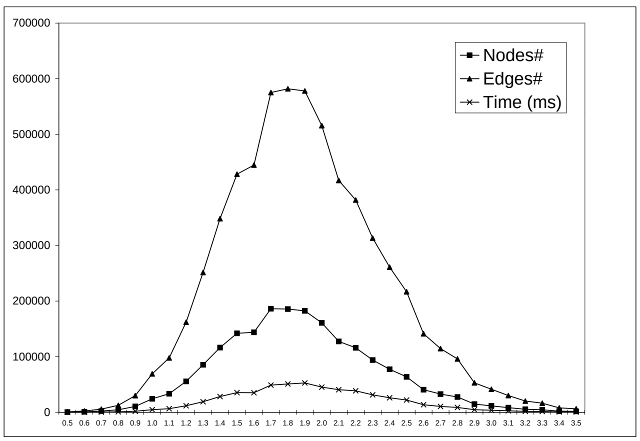

Figure 4: Difficulty of compilation according to clauses/vars ratio. Each point is the average over 100 instances.

second in this case. Hence, the compilation time is amortized over all queries which makes the time-to-compile a worthy investment in this case. We note here that the first instance of flat100 could not be compiled into an OBDD using CUDD within a cutoff time of 1 hour. One can count the models of flat100 efficiently however using the RELSAT[10] model counter, but we report in the following section on other CNFs which could not be handled efficiently using RELSAT.

We also experimented with planning CNFs from SATLIB. We could compile blocks-world CNFs anomaly, medium, huge, and large.a within a few minutes each. But we could not compile large.b, nor the logistics CNFs within a few hours.

We close this section by noting that random 3CNF formulas in the crossover region, those with clauses/vars ratio of about 4.3, are easier to compile than formulas with lower ratios. The same has been observed for counting models, where the greatest difficulty is reported for ratios around 1.2 by (Birnbaum & Lozinskii 1999) and around 1.5 by (Bayardo & Pehoushek 2000). Figure 4 plots information about compilations of random 3CNFs with 50 variables each, for clauses/vars ratio ranging from $.5$ to 3.5 at increments of $.1$. As is clear from this plot, the peak for the number of nodes, number of edges, and time is around a ratio of 1.8.

[10] *http://www.almaden.ibm.com/cs/people/bayardo/vinci/index.html*

Boolean Circuits

We now consider CNFs which correspond to digital circuits. Suppose we have a circuit with inputs I, outputs O and let W stand for all wires in the circuit that are neither inputs nor outputs. We will distinguish between three types of representations for the circuit:

Type I representation: A theory Δ over variables I, O, W where the models of Δ correspond to instantiations of I, O, W that are compatible with circuit behavior. A CNF corresponding to Type I representation can be easily constructed and in a modular way by generating a set of clauses for each gate in the circuit.[11]

Type II representation: A theory Δ over input/output variables I, O, where the models of Δ correspond to input/output vectors compatible with circuit behavior. If Δ is a Type I representation, then $\exists W \Delta$ is a Type II representation.[12]

Type III representation for circuit output o: A theory over inputs I, where the models correspond to input vectors that generate a 1 at output o. If Δ is a Type II representation, then $\exists o.\Delta \wedge o$ is a Type III representation for output o.

Clearly, Type I is more expressive than Type II, which is more expressive than Type III. The reason we draw this distinction is to clarify that in the formal verification literature, one usually constructs Type III representations for circuits since this is all one needs to check the equivalence of two circuits. In AI applications, however, such as diagnosis, one is mostly interested in Type I representations, which are much harder to obtain.

We compute Type II representations by simply replacing Lines 12 & 15 in CNF2DDNNF by

$$\alpha^+ \leftarrow \text{CONJOIN}(\Pi', \text{CASE\_ANALYSIS}(n, \Pi \cup \Omega)),$$

[11] Type I representations are called *Circuit Consistency Functions* in (Aloul, Markov, & Sakallah 2001; December 2001).

[12] Recall: $\exists w \Delta$, where w is a single variable, is defined as $\Delta^+ \vee \Delta^-$, where Δ^+ (Δ^-) is the result of replacing w with $true$ ($false$) in Δ. $\exists W \Delta$ is the result of quantifying over variables in W, one at a time (Darwiche & Marquis 2001).

and

$$\alpha^- \leftarrow \text{CONJOIN}(\Pi', \text{CASE\_ANALYSIS}(n, \Pi \cup \Omega)),$$

respectively, where Π' is obtained from Π by removing all literals corresponding to variables in W. We also have to modify the boundary condition handled by CLAUSE2DDNNF, so that CLAUSE2DDNNF(.) returns *true* if the clause . contains a literal pertaining to W, and behaves as usual otherwise. Given the above changes—which implement the proposal given in (Darwiche 2001a) for existential quantification—CNF2DDNNF$(n, \emptyset)$ is then guaranteed to return $\exists W \Delta$ in d-DNNF, where n is the root of a dtree for CNF Δ.[13]

To compute efficient Type III representations, one needs to use multi-rooted NNFs, where each root corresponds to the compilation of one circuit output. This is how it is done in the formal verification literature, where multi-rooted OBDDs are known as *shared OBDDs*. Our compiler does not handle multi-rooted d-DNNFs yet, so we do not report on Type III representations.

Tables 2 and 3 contain results on the first five circuits in the ISCAS85 benchmark circuits.[14] We were able to obtain Type I and Type II representations for all these circuits expressed as d-DNNFs. The most difficult was c1908, which took around 1.5 hrs, followed by c880 which took around 30 minutes. We are not aware of any other compilations of these circuits of Types I and II, although the formal verification literature contains successful compilations of Type III, represented as multi-rooted OBDDs. We could not compile c499, c880, c1355, nor c1908 into Type I OBDDs using CUDD, nor could we count their models using RELSAT, within cutoff times of 1hr, 1hr, 1hr and 3hrs, respectively (we actually tried CUDD on c499 for more than a day). For c432, we tried several OBDD ordering heuristics. The best OBDD we could obtain for this circuit had 15811 nodes.

We note here that although d-DNNF does not support a deterministic test of equivalence, one can easily test the equivalence of a d-DNNF Δ, and a CNF $\Gamma = \gamma_1 \wedge \ldots \wedge \gamma_m$, which corresponds to a Type I representation of a circuit. By construction, the number of models for Γ is 2^k, where k is the number of primary inputs for the circuit. Therefore, Δ and Γ are equivalent iff (1) the number of models for Δ is 2^k and (2) $\Delta \models \Gamma$. The first condition can be checked in time linear in the size of Δ since d-DNNF supports model counting in linear time. The second condition can be checked by verifying that $\Delta \models \gamma_i$ for each i, a test which can also be performed in time linear in the size of Δ since d-DNNF supports a linear test for clausal entailment. We actually use the above technique for checking the correctness of our d-DNNF compilations.

Table 4 contains further results from ISCAS89.[15] These are sequential circuits, which have been converted into combinational circuits by cutting feedback loops into flip-flops, treating a flip-flop's input as a circuit output and its output as a circuit input. Most of these circuits are easy to compile and have relatively small d-DNNFs. Type I OBDD representations for some ISCAS89 circuits are reported in (Aloul, Markov, & Sakallah 2001; December 2001), which is probably the most sophisticated approach for converting CNFs into OBDDs. In addition to proposing a new method for ordering OBDD variables based on the connectivity of given CNF, a proposal is made for ordering the clauses during the OBDD construction process. (Aloul, Markov, & Sakallah 2001; December 2001) report on the maximum number of OBDD nodes during the construction process, not on the size of final OBDDs constructed. Yet, their experiments appear to confirm the theoretical results reported in (Darwiche & Marquis 2001) on the relative succinctness of d-DNNF and OBDD representations. For example, circuits s832, s953, s1196 and s1238 were among the more difficult ones in these experiments, leading to constructing 115×10^3, 1.8×10^6, 2×10^6, and 2×10^6 nodes, respectively—s1238 is the largest circuit they report on. These numbers are orders of magnitude larger than what we report in Table 4.[16] We note here that the total number of nodes constructed by our d-DNNF compiler is rarely more than twice the number of nodes in the final d-DNNF.[17] We finally note that no experimental results are provided in (Aloul, Markov, & Sakallah 2001;

Name	Vars/Clause	d-DNNF nodes	d-DNNF edges	Clique size	Time (sec)
c432	196/514	2899	19779	28	6
c499	243/714	691803	2919960	23	448
c880	443/1112	3975728	7949684	24	1893
c1355	587/1610	338959	3295293	23	809
c1908	913/2378	6183489	12363322	45	5712

Table 2: Type I compilations of ISCAS85 circuits.

Name	I/O vars	d-DNNF nodes	d-DNNF edges	Time (sec)
c432	36/7	952	3993	1
c499	41/32	68243	214712	127
c880	60/26	718856	2456827	1774
c1355	41/32	65017	201576	483
c1908	33/25	326166	1490315	4653

Table 3: Type II compilations of ISCAS85 circuits.

[13] In general, this only guarantees that the result is in DNNF (Darwiche 2001a). For CNFs corresponding to digital circuits, however, determinism is also guaranteed due to the following property: for every instantiation α of I, O, there is a unique instantiation β of W such that $\Delta \wedge \alpha \models \beta$.

[14] *http://www.cbl.ncsu.edu/www/CBL_Docs/iscas85.html*

[15] *http://www.cbl.ncsu.edu/www/CBL_Docs/iscas89.html*

[16] One has to admit though that it is hard to tell exactly how much of this difference is due to relative succinctness of OBDD vs d-DNNF, and how much of it is due to the effectiveness of different compilation techniques, since none of the compilers discussed are guaranteed to generate optimal OBDDs or d-DNNFs.

[17] This is in contrast to OBDD compilers, where the number of intermediate OBDD nodes can be much larger than the size of final OBDD returned. We believe this is due to the top-down construction method used by our compiler, as opposed to the bottom-up methods traditionally used by OBDD compilers.

Name	Vars/Clause	d-DNNF nodes	d-DNNF edges	Clique size	Time (sec)
s298	136/363	830	4657	12	1
s344	184/429	962	4973	9	1
s349	185/434	1017	5374	10	1
s382	182/464	1034	5081	17	1
s386	172/506	1401	10130	21	2
s400	186/482	1021	5137	18	1
s444	205/533	1091	5872	16	1
s499	175/491	1090	5565	20	2
s510	236/635	967	5755	38	2
s526	217/638	2621	19605	22	1
s526n	218/639	2611	20115	22	1
s635	320/762	1360	4845	9	1
s641	433/918	7062	84596	21	1
s713	447/984	7128	90901	21	10
s820	312/1046	2774	21365	29	2
s832	310/1056	2757	21224	28	2
s938	512/1233	2207	12342	14	2
s953	440/1138	11542	110266	64	14
s967	439/1157	20645	443233	60	117
s991	603/1337	2382	13107	8	2
s1196	561/1538	12554	261402	51	60
s1238	540/1549	14512	288143	53	58
s1423	748/1821	112701	1132322	24	162
s1488	667/2040	6338	62175	49	11
s1494	661/2040	6827	64888	51	12
s1512	866/2044	12560	140384	21	27
s3330	1961/4605	358093	8889410	43	5853
s3384	1911/4440	44487	392223	17	45

Table 4: Type I compilations of ISCAS89 circuits.

December 2001) for Type I OBDD representations of ISCAS85 circuits, which are much harder to compile than ISCAS89 circuits.

One implication of our ability to compile these circuits is that we can now perform a variety of reasoning tasks about these circuits in time linear in the size of given d-DNNF. Some example queries: Given a distribution over the circuit inputs, what is the probability that Wire 45 is high? How may circuit inputs will generate a high on the first circuit output and a low on the fifth output? Is it true that whenever Wires 33 and 87 are high, then Wire 19 must be low? How may input vectors will generate a particular output vector? Each one of these queries can be answered by a single traversal of the d-DNNF circuit representation (Darwiche 2001b; 2001a; 2001c).

We also report in Tables 2–4 on the best clique sizes obtained for these circuits when converting their structures into jointrees (Jensen, Lauritzen, & Olesen 1990). This is needed for reasoning about these circuits probabilistically using state-of-the-art algorithms for Bayesian networks. These algorithms have exponential complexity in the clique size. Hence, most of these circuits are outside the scope of such algorithms. We are not aware of any algorithm for probabilistic reasoning which can handle these circuits, except the one we report on in (Darwiche 2001b) which is based on these d-DNNF compilations. We close this section by noting that the algorithm reported in (Darwiche 2001a; 2001c) also has a time complexity which is exponential in the clique size. Hence, most of the CNFs we considered in this paper are outside the scope of the mentioned algorithm.

Relationship to Davis-Putnam

One cannot but observe the similarity between our proposed algorithm and the Davis-Putnam (DP) algorithm for propositional satisfiability (Davis, Logemann, & Loveland 1962), and its recent extensions for counting propositional models: the CDP algorithm in (Birnbaum & Lozinskii 1999) and the DDP algorithm in (Bayardo & Pehoushek 2000).

The DP algorithm solves propositional satisfiability by performing case analysis until a solution is found or an inconsistency is established. When performing case analysis on variable X, the second value for X is considered only if the first value does not lead to a solution. The CDP algorithm in (Birnbaum & Lozinskii 1999) observed that by always considering both values, we can extend the DP algorithm to count models since $ModelCount(\Delta) = ModelCount(\Delta^+) + ModelCount(\Delta^-)$, where Δ^+ and Δ^- are the result of setting X to $true$ and to $false$, respectively, in Δ. The DDP algorithm in (Bayardo & Pehoushek 2000) incorporated yet another idea: If Δ can be decomposed into two disconnected subsets Δ^1 and Δ^2, then $ModelCount(\Delta) = ModelCount(\Delta^1) ModelCount(\Delta^2)$. Hence, DDP will apply case analysis until the CNF is disconnected into pieces, in which case each piece is attempted independently.

The CDP algorithm can in fact be easily adapted to compile a CNF into a d-DNNF, by simply constructing the NNF fragment $X \wedge \text{CDP}(\Delta^+) \vee \neg X \wedge \text{CDP}(\Delta^-)$ each time a case analysis is performed on variable X. Here, $\text{CDP}(.)$ is the result of compiling . into d-DNNF using the same algorithm recursively. This extension of CDP will generate a strict subset of d-DNNF: the one which satisfies the decision and decomposability properties (hence, an FBDD) and that also has a tree structure (FBDDs have a graph structure in general). FBDDs are known to be less succinct than d-DNNFs, even in their graph form (Darwiche & Marquis 2001). The tree-structured form is even more restrictive.

The DDP algorithm can also be easily adapted to compile a CNF into a d-DNNF, by constructing the NNF fragment $X \wedge \text{DDP}(\Delta^+) \vee \neg X \wedge \text{DDP}(\Delta^-)$ each time a case analysis is performed on X, and by constructing the fragment $\text{DDP}(\Delta^1) \wedge \text{DDP}(\Delta^2)$ each time a decomposition is performed as given above. This extension of DDP will actually generate d-DNNFs which are not FBDDs, yet are still tree-structured which is a major limitation. The important point to stress here is that any CNF which can be processed successfully using the DDP algorithm, can also be compiled successfully into a d-DNNF.

The algorithm we present can be viewed as a further generalization of the discussed DDP extension in the sense that it generates graph NNFs as opposed to tree NNFs. The graph structure is due to two features of CNF2DDNNF: the caching and unique-node schemes. Each time a node is looked up from a cache, its number of parents will potentially increase by one. Moreover, the CONJOIN and DISJOIN operations will often return a pointer to an existing NNF node instead of constructing a new one, again, increasing the number of parents per node.[18] Another major difference with the above proposed extension of DDP is the use of dtrees to guide the decomposition process as they restrict the set of variables considered for case analysis at any given time. The use of dtrees can then be viewed as a variable splitting heuristic which is geared towards decomposition as opposed to solution finding.

Conclusion

We presented a compiler for converting CNF formulas into deterministic, decomposable negation normal form (d-DNNF). This is a logical form that has been identified recently and shown to support a number of operations in polynomial time, including clausal entailment; model counting, minimization and enumeration; and probabilistic equivalence testing. d-DNNFs are also known to be a superset of, and more succinct than, OBDDs. The logical operations supported by d-DNNFs are a subset of those supported by OBDDs, yet are sufficient for model-based diagnosis and planning applications. We presented experimental results on compiling a variety of CNF formulas, some generated randomly and others corresponding to digital circuits. A number of the formulas we were able to compile efficiently could not be similarly handled by some state-of-the-art model counters, nor by some state-of-the-art OBDD compilers. Moreover, our ability to successfully compile some of these CNFs allowed us to answer some queries for the very first time.

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[18](Bayardo & Pehoushek 2000) rightfully suggest that "learning goods," which corresponds to caching non-zero counts, is essential for efficient counting of models, but do not pursue the technique citing technical difficulties.

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